* Simple stress and strains

Loads :-

A Load may be defined as the combined effect of external forces acting on a body.

The loads may be classified as

(i) Dead Load
(ii) Live Load
(iii) Inertia Loads
(iv) Centrifugal Loads / forces

The other way of classification is

(i) Tensile Loads
(ii) compressive Loads
(iii) Torsional / Twisting Loads
(iv) Bending Loads
(v) shearing loads.

* On basis of its nature :-

Based on of its nature Loads are classified as

Dead Load
Live Load
Wind Load
Snow Load
Seismic Load

Dead Load :- These Loads are permanent and remain in place throughout the life of structure.

eg:- self weight of structure.

Live Load :- These Loads are not permanent and are movable throughout the life of the structure.

eg:- human beings, furniture.

Wind Load :- These Loads are applied by wind pressure on a structure.

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Snow Load:- These Loads are applied by accomodation of snow over the structure.

Seismic Load:- These are the Loads which causes during an earthquake.

* on basis of its fixidity:

(a) Static Load:- These Lods remain nearly constant with time.

eg:- dead Load, floor load.

(b) Dynamic Load:- These Loads don't remain constant that is they vary with time.

eg:- live Load.

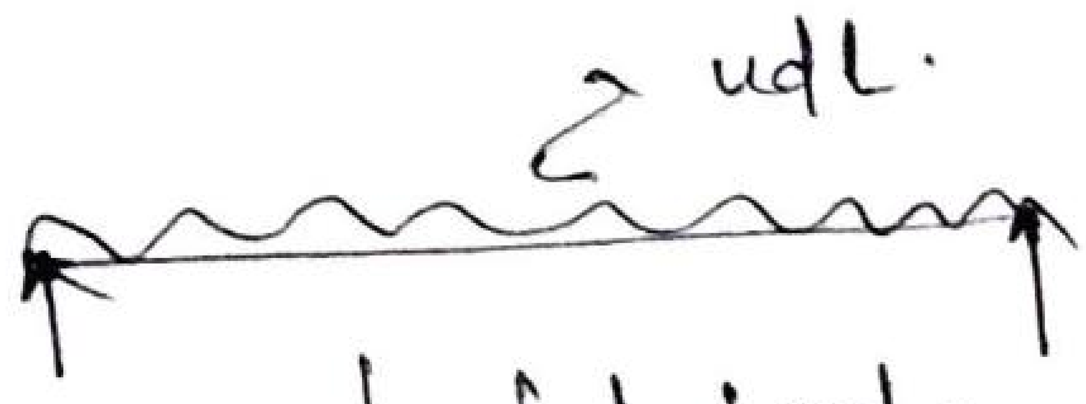
* on basis of its area of application

(a) Distributed Load:- These Loads are distributed equally or unequally over a perticular surface length or area of a member.

eg:- Load from slab to beam.

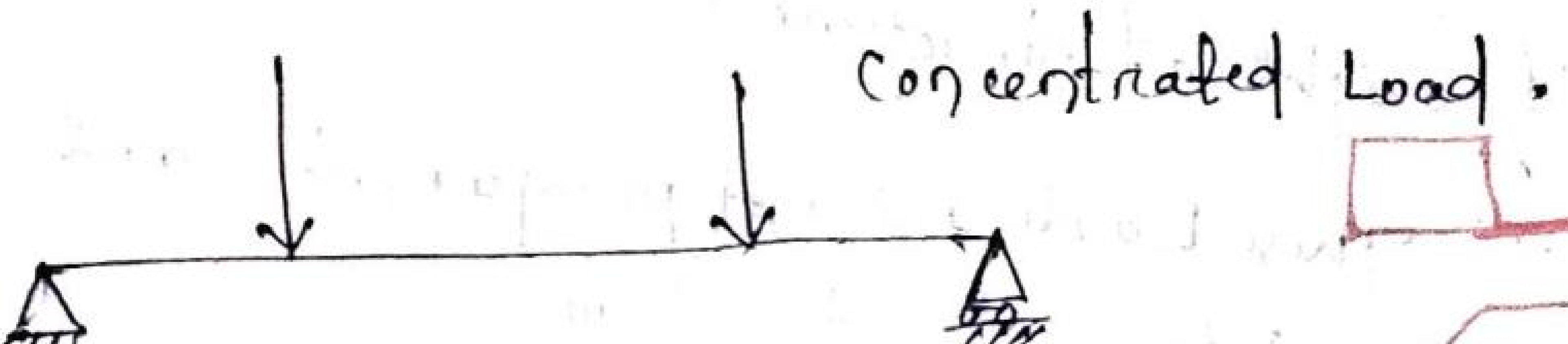
Slab - 20 mm
↓
Beam - 25
↓
column - 40
↓
foundation - 50

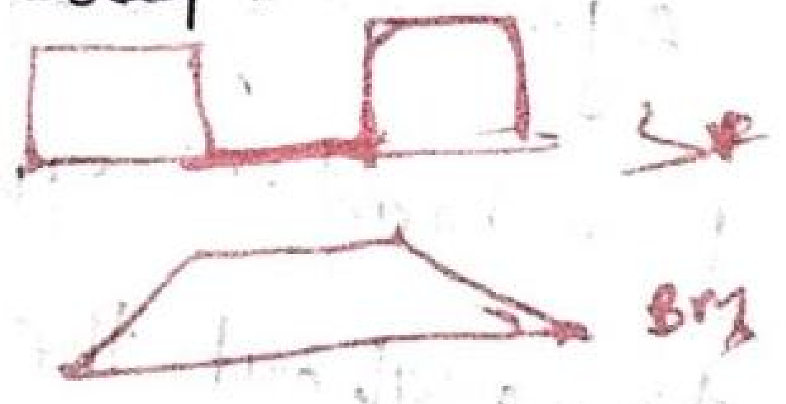
clear cover



(b) concentrated Load:- These Loads are applied on a small contact area or at a point on a member.

eg:- point Load on beam.

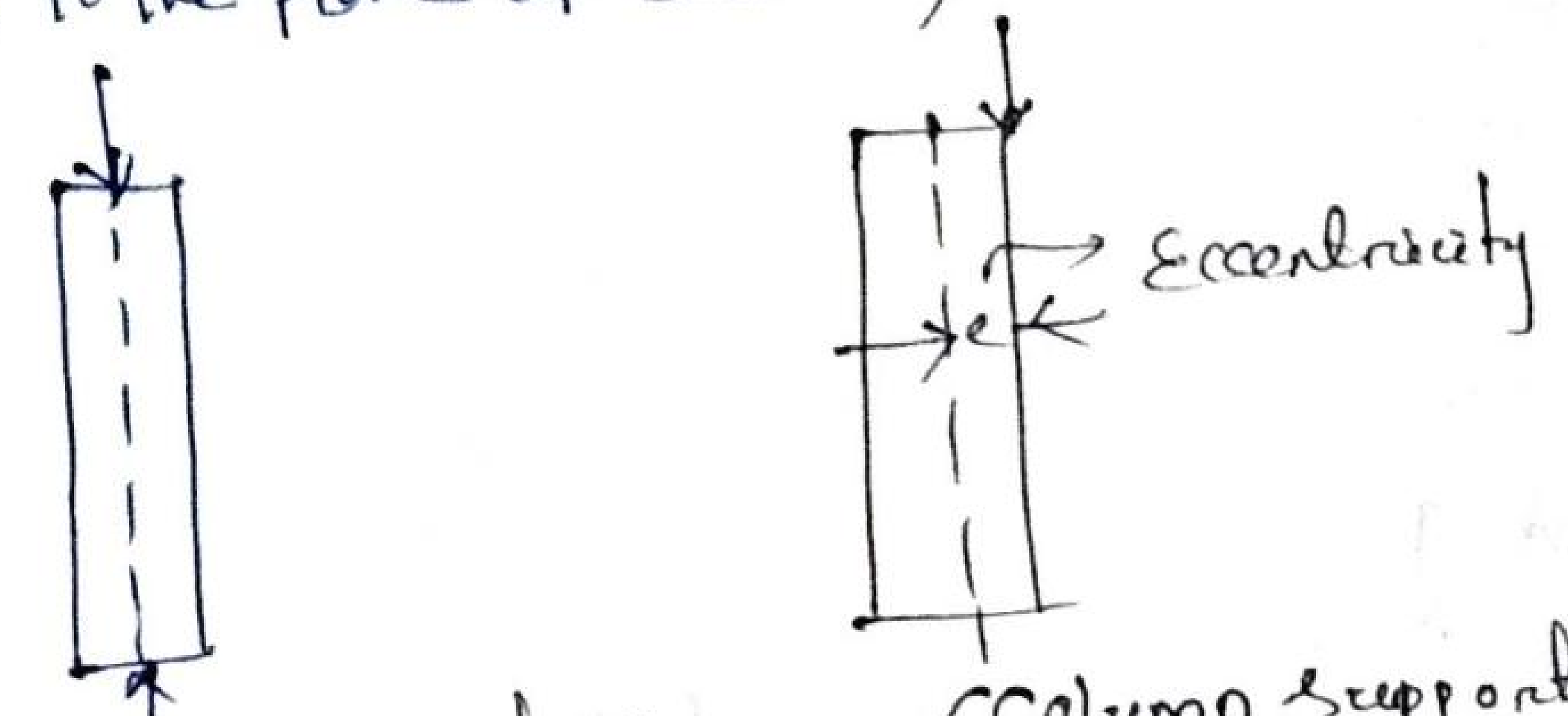




* on basis of nature of application

(a) Axial Load:- The force whose resultant passes through the centroid of a section and its ⊥r to the plane of section.

(b) Eccentric Load :- The force whose resultant doesn't pass through the centroid of a section and it is $\perp^r$ to the plane of section.



Column supporting axial Load

(Column supporting eccentric Load)

* Stress :- :- It is the internal response to external force.

When a body is acted upon by some Load or external force it undergoes deformation. (change in shape or dimensions)

→ (The internal resistance which the body offers to meet with the Load is called stress).

→ The resistance per unit area to deformation is k/w as stress.

Mathematically stress may be defined as force per unit area i.e stress.

$$\boxed{\sigma = P/A}$$

P = Load or force acting on the body (KN/N)

A = C/s area of the body. (m^2/mm^2)

* Stress can be considered either as total stress or unit stress.

Total stress = (N, KN, MN)

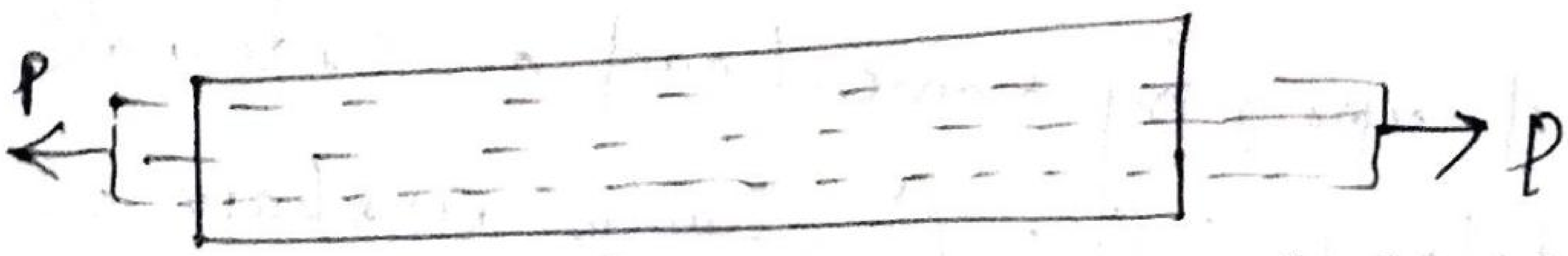
It represent the total resistance to an external effect

unit stress represent resistance developed by unit area.

represented by (KN/m^2, MN/m^2).

* The various types of stresses may be classified as
(1) simple or direct stress
    (i) Tension
    (ii) compression
    (iii) shear.
(2) Indirect stress
    (i) Bending
    (ii) Torsion.
(3) combined stress :- combination of type 1 & 2

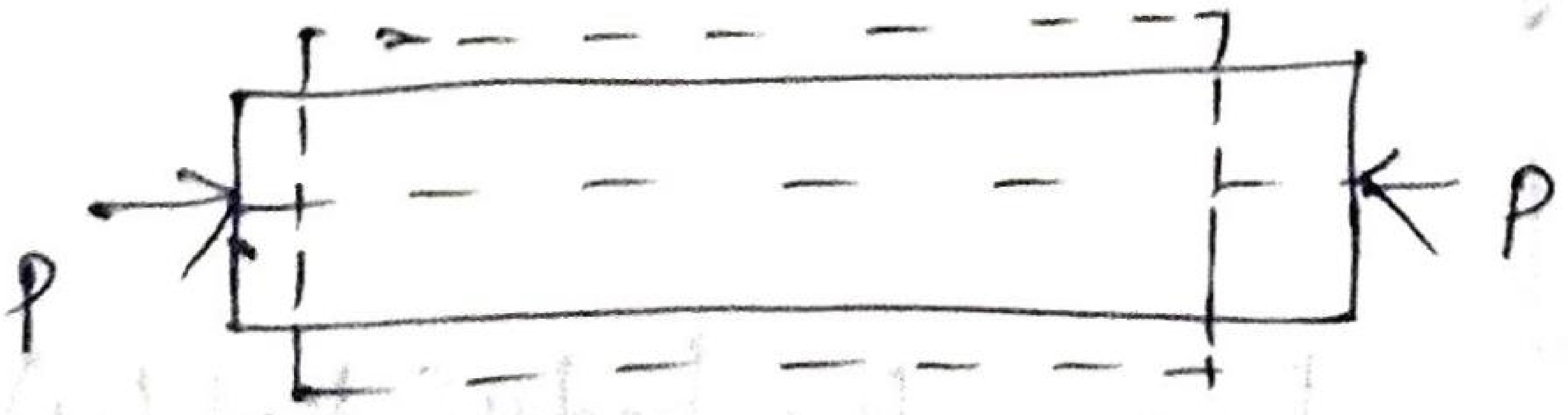
* Tensile stress :-



(Tensile stress)

When a section is subjected to two equal and opposite pulls and the body tends to increase its length the stress induced is called tensile stress.

* compressive stress :- When a section is subjected to two equal and opposite pushes and the body tends to shorten in length. the stress induced is called compressive stress.



(Compressive stress)

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* Beyond the elastic limit, the material gets i plastic stage and in this stage the deformation doesn't entirely disappear on the removal of force.

* **Strain**

Any element in a material subjected to stress is said to be strained.

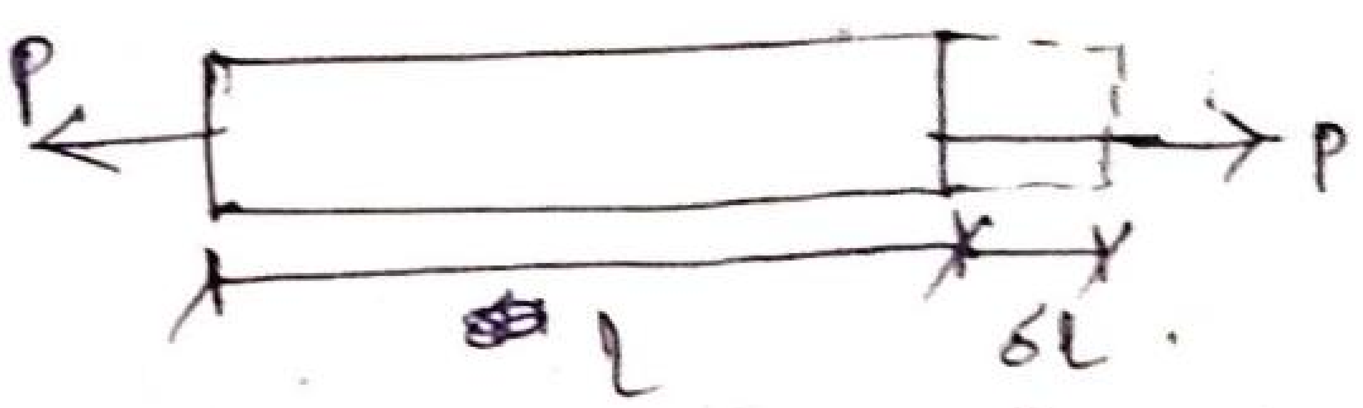
→ Strain(e) is the deformation produced by stress.

Tensile strain:-

A piece of material with uniform c/s subjected to a uniform axial tensile stress will increase its length from L to (L+δL)

→ The increament of length δL is the actual deformation of the material.

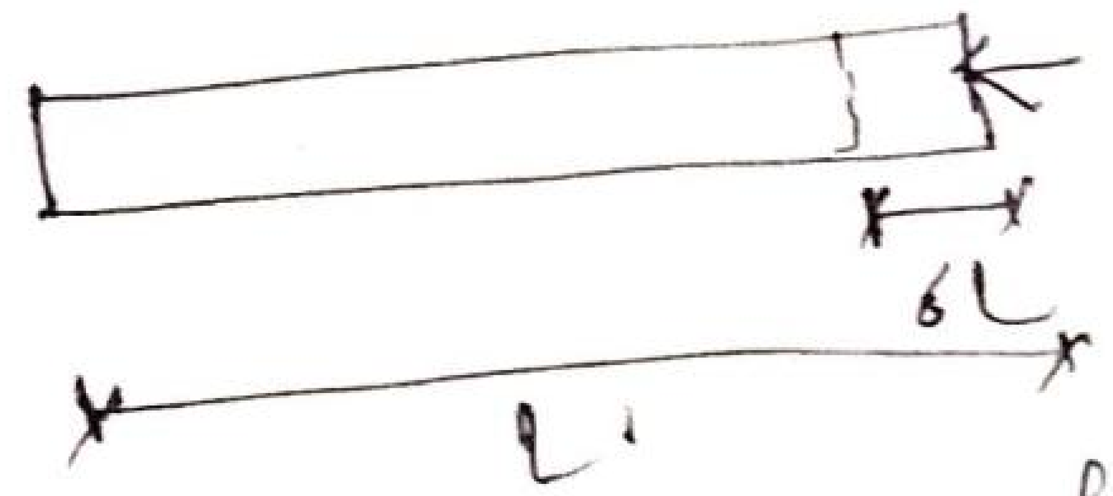
$$\boxed{e_t = \frac{\delta L}{L}}$$



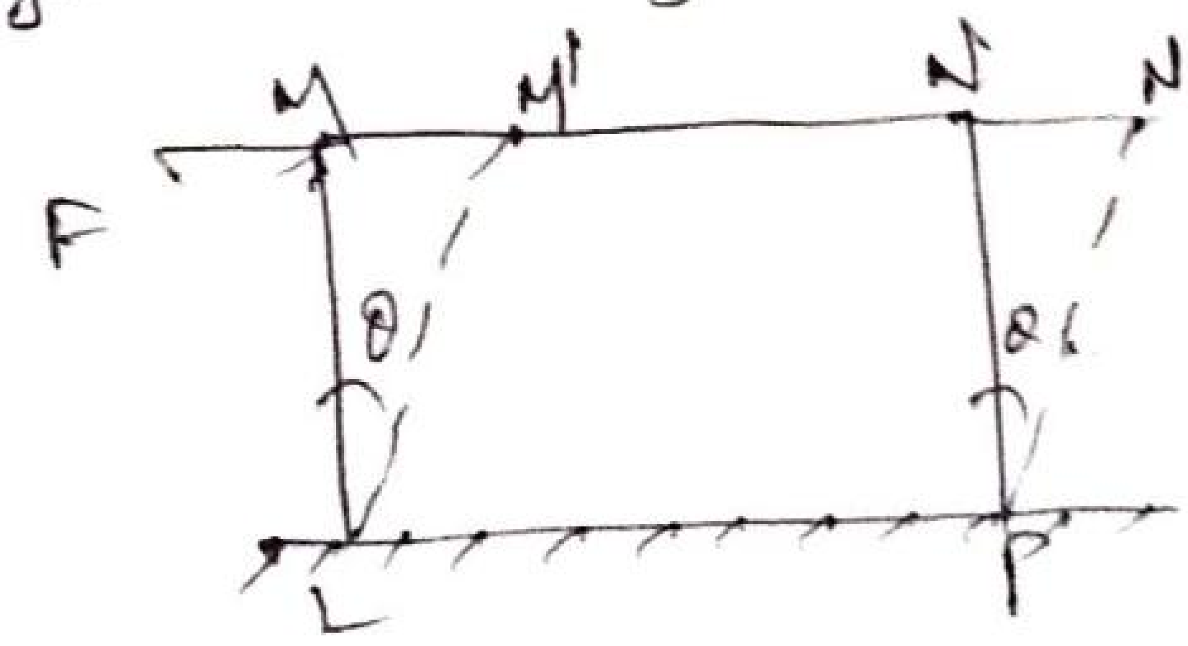
Compressive strain:-

Under compressive forces a similar piece of material would be reduced in length.

$$e_c = \delta L/L$$



Shear strain:- In case of shearing load, a shear strain will be produced which is measured by the angle through which body distorts



$$e_s = \frac{NN'}{NP} = \tan\phi$$
$$= \phi \text{ (radians)}.$$

[Properties of Materials]

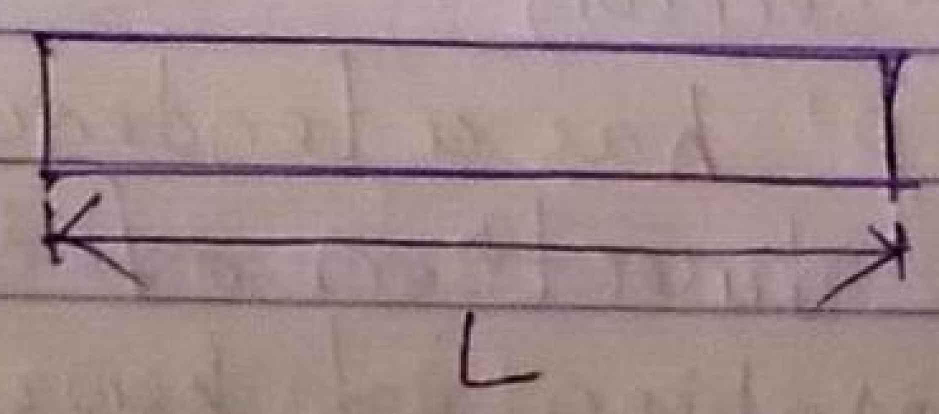
Introduction

It is a branch of applied Mechanics that deals with the behaviour of solid bodies subjected to various types of loading and internal forces developed due to these Loading.

→ The objective of our analysis is will be to determine the stresses, strains and deflections produced by the Loads in different structures.

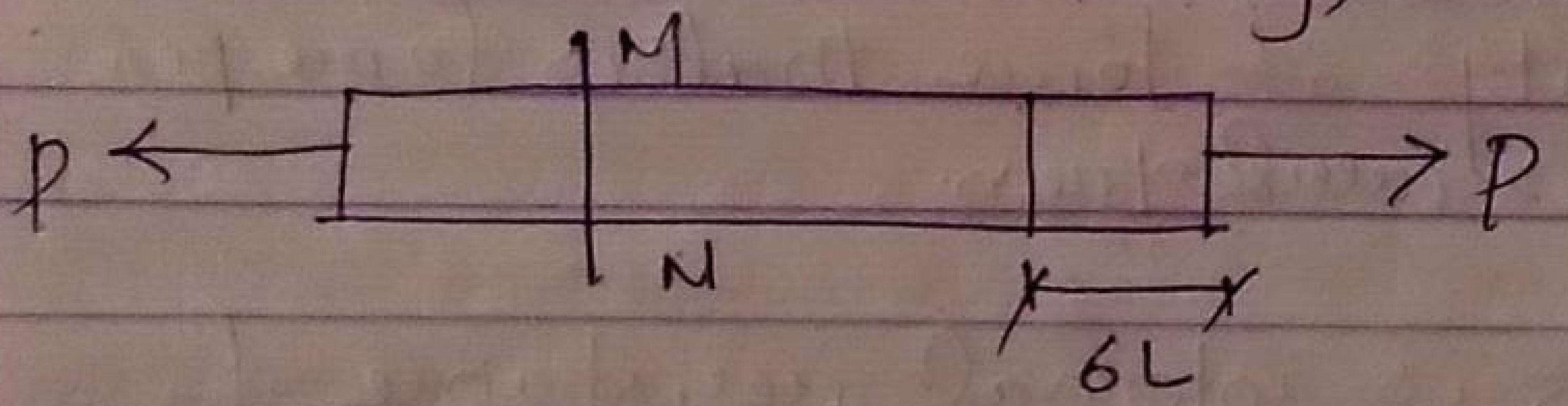
Normal stress

Consider a prismatic bar of length 'L' and that is loaded by axial force 'p' at the end as shown in figure



(a)

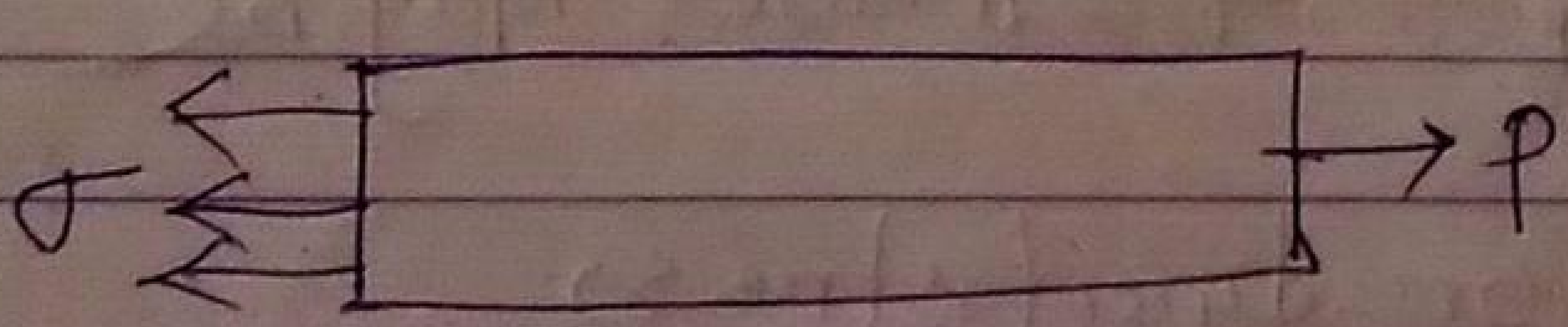
It is a straight structural member having constant crossectional area throughout its length.



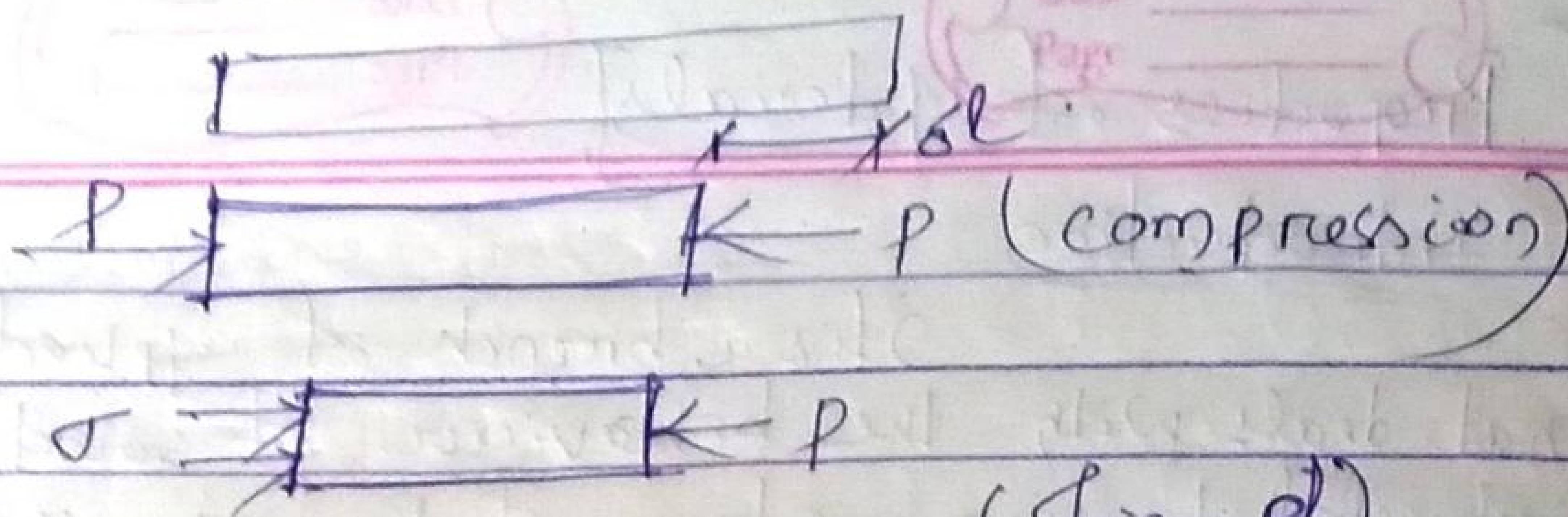
(Tension)

fig-b

In figure (b) axial force produce a uniform stretching of the bar hence the bar is said to be in tension.



(fig-c)



(fig - d)

In figure (c) force produce uniform compression of the bar. hence the bar is said to be in compression.

To investigate the internal stresses produced in the bar by axial forces we make an imaginary cut at section MN. This section is taken perpendicular to the Longitudinal axis of bar hence it is known as cross-section.

The force 'p' has a tendency to move free body in the direction of Load. So to restrict the motion of bar an internal force is induced which is uniformly distributed over cross-sectional area. The intensity of force that is force per unit area is called stress.

* Stress is internal resistance of material offered against deformation which is force per unit area.

* The various types of stresses may be classified as
  (1) simple or direct stress
  - → Tension
  - → compression
  - → shear

(2) Indirect stress
→ Bending stress
→ Torsion.

Note :- Stress induced in Material depends upon the nature of force, point of application and C/s area of Material.

→ Stress can be Tensile or compressive. represented by (σ). Mathematically $\boxed{\sigma = P/A}$

unit - N/mm^2 or MPa

Sign convention

Tensile = +ve

Comp. = -ve.

* Stresses are induced only when Motion of bar is restricted either by some force or reaction induced. If body or bar is free to Move or expansion is allowed then no stresses will be induced.

→ Pressure has same unit but pressure is different physical quantity than stress. pressure is external normal force distributed over surface.

On the basis of C/s area considered stress is of 2 types.

(1) Engineering / Nominal stress

(2) True / Actual stress.

Engineering stress :-

$$\boxed{\sigma = P/A_0}$$

P = force
A_0 = original c/s area.

True stress / Actual stress :-

$$\boxed{\sigma = P/A_a}$$

A_a = Actual c/s area of specimen at Loading.

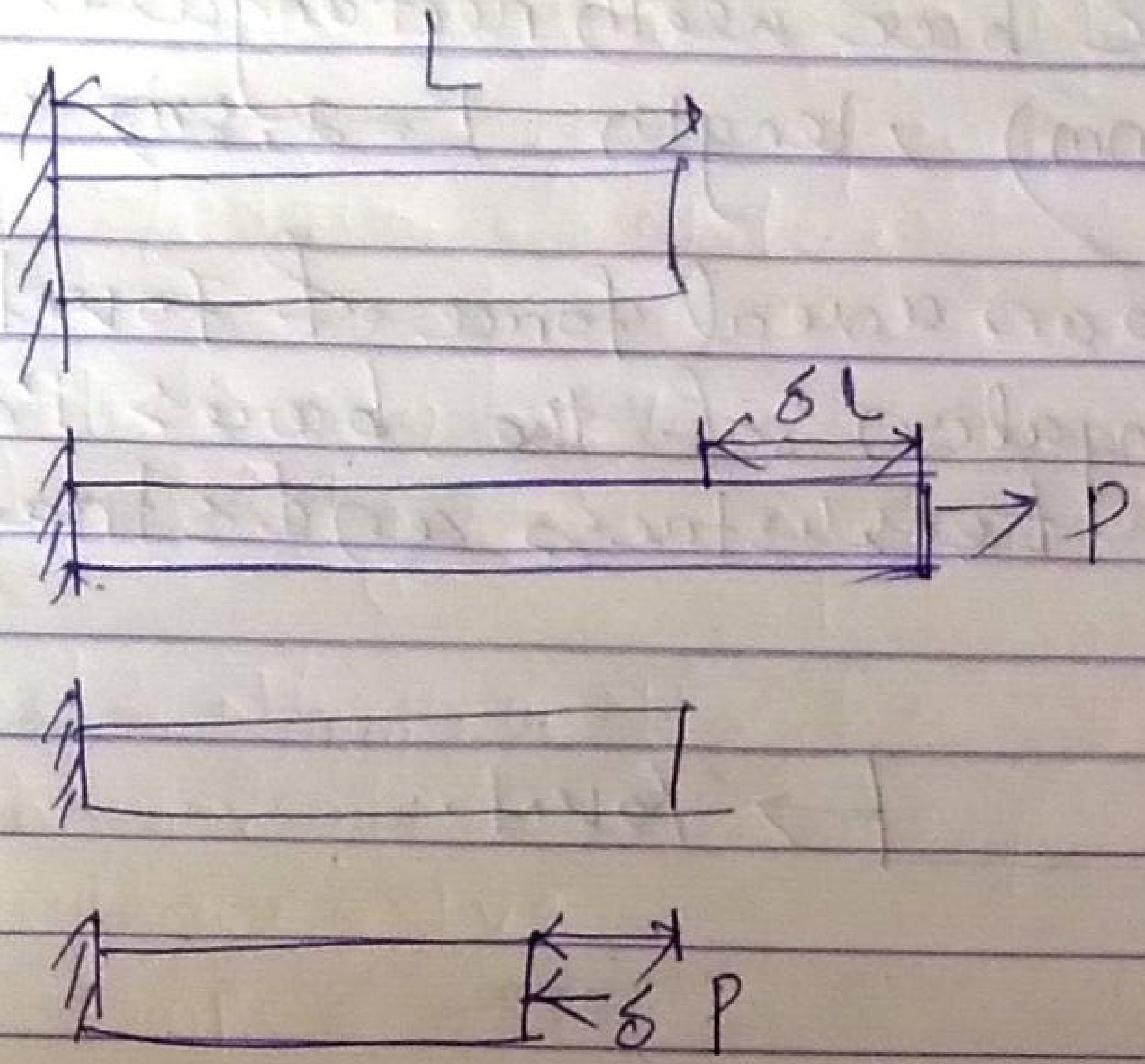
Strain :-

An axially loaded bar undergoes a change in length becoming longer when in tension and shorter when in compression and is represented by εore.

Mathematically $\epsilon = \delta / L$

$= \frac{\text{change in length}}{\text{original length}}$.

unit :- Strain is dimension less quantity.



On the basis of length of member used in calculation strain can be following 2 types.

(1) Engg. or Nominal strain

(2) True / Actual strain.

Engg / Nominal strain :-

$$\boxed{\epsilon_0 = \Delta l / l_0}$$

l_0 = original length

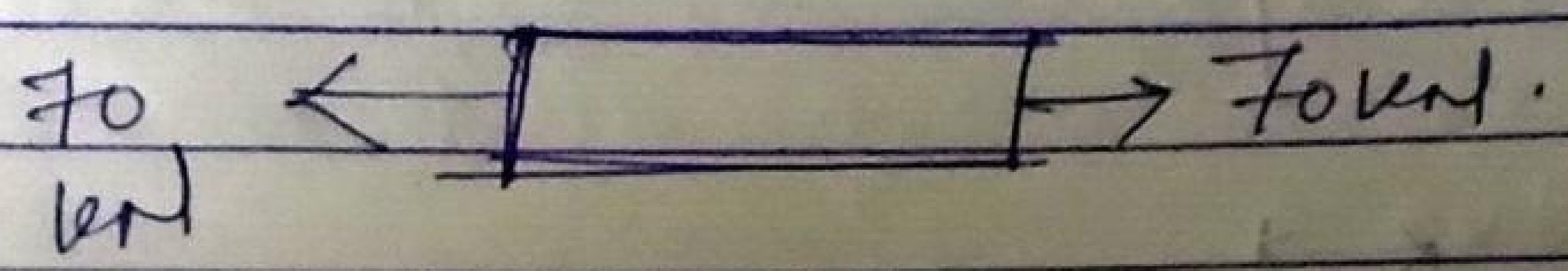
Δl = change in length

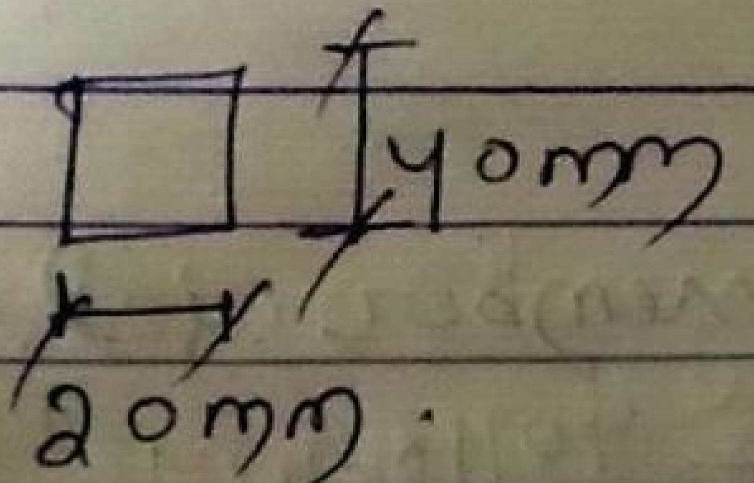
True / Actual strain

When length of member is taken as actual length of member at loading.

$$\boxed{\epsilon_a = \Delta y / l_a}$$

Q. A prismatic bar with rectangular c/s (20mm x 40mm), length L = 2.8m is subjected to an axial force of 70kN. The measured elongation of the bar is 1.2mm. Calculate tensile stress and strain in bar.





Ans:- Assuming that force at C.G. of section.

We k/w that

$$\text{Stress}(\sigma) = P/A = \frac{70\times10^3}{20\times40} = 87.5\,MPa$$

$$\text{Strain} = \Delta y / L_0 = \frac{1.2}{2.8\times1000} = 4.285\times10^{-4}$$

Simple stress- Strain and Elastic constants

Stress :- Stress is internal resistance per unit area offered by material against deformation.

→ Its unit is N/mm^2 or MPa.

Depending upon nature of stress, stress may be following types.

1) Normal stress
2) Shear stress.

Normal stress :-

Normal stresses are always normal to the cross section at any section.

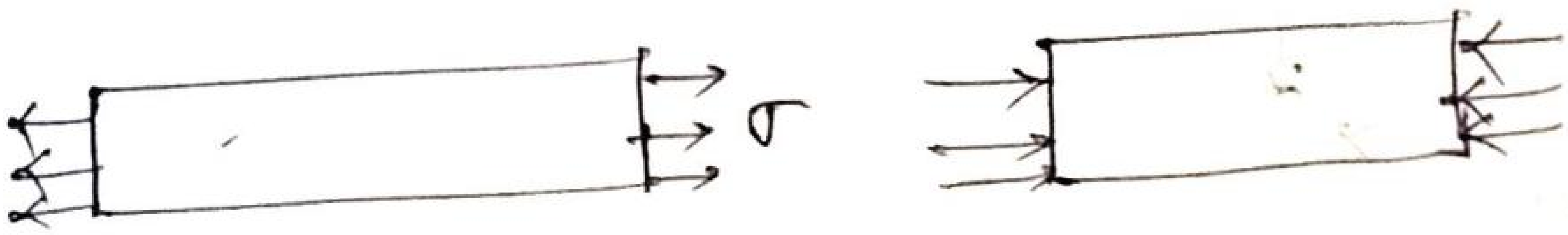
→ It is represented by (σ), unit is N/mm^2.

Normal stress is of 2 types.

(a) **Direct / Axial stress :-** These stresses are produced when an axial force is acted at C.G of C/s.

→ For prismatic body with axial loading direct stresses are uniform across the C/s.

Generally tensile stresses are taken as positive and comp. stresses are taken as -ve.

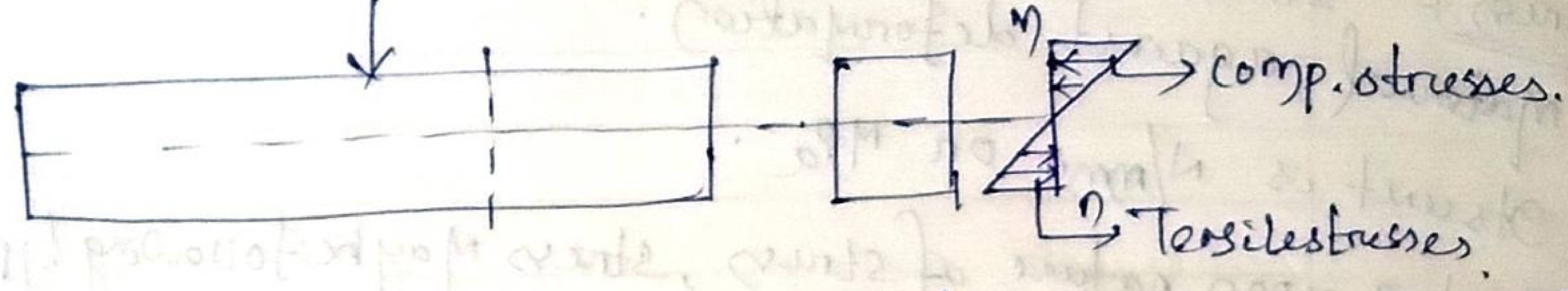


b) **Bending stresses :-**

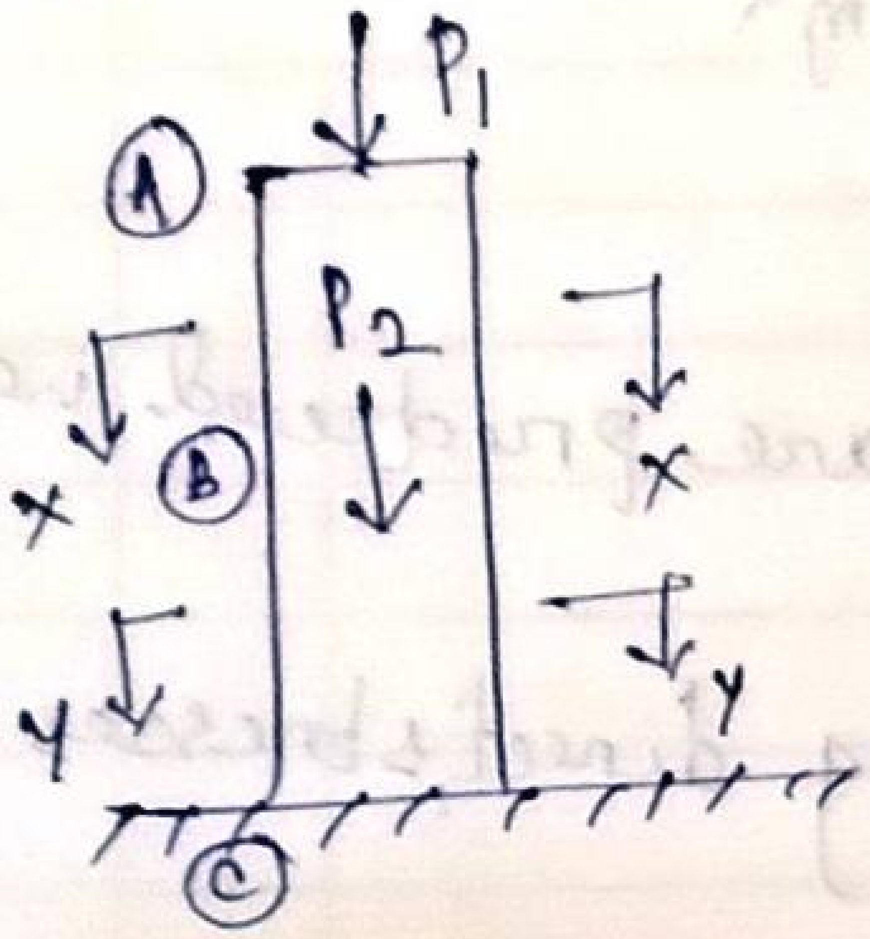
Bending stresses are produced by bending moment.

Bending stresses varies linearly from zero at N.A to maximum at farthest fibre from N.A.

→ Tensile bending stresses are taken as positive and compressive bending stresses are taken as −ve.



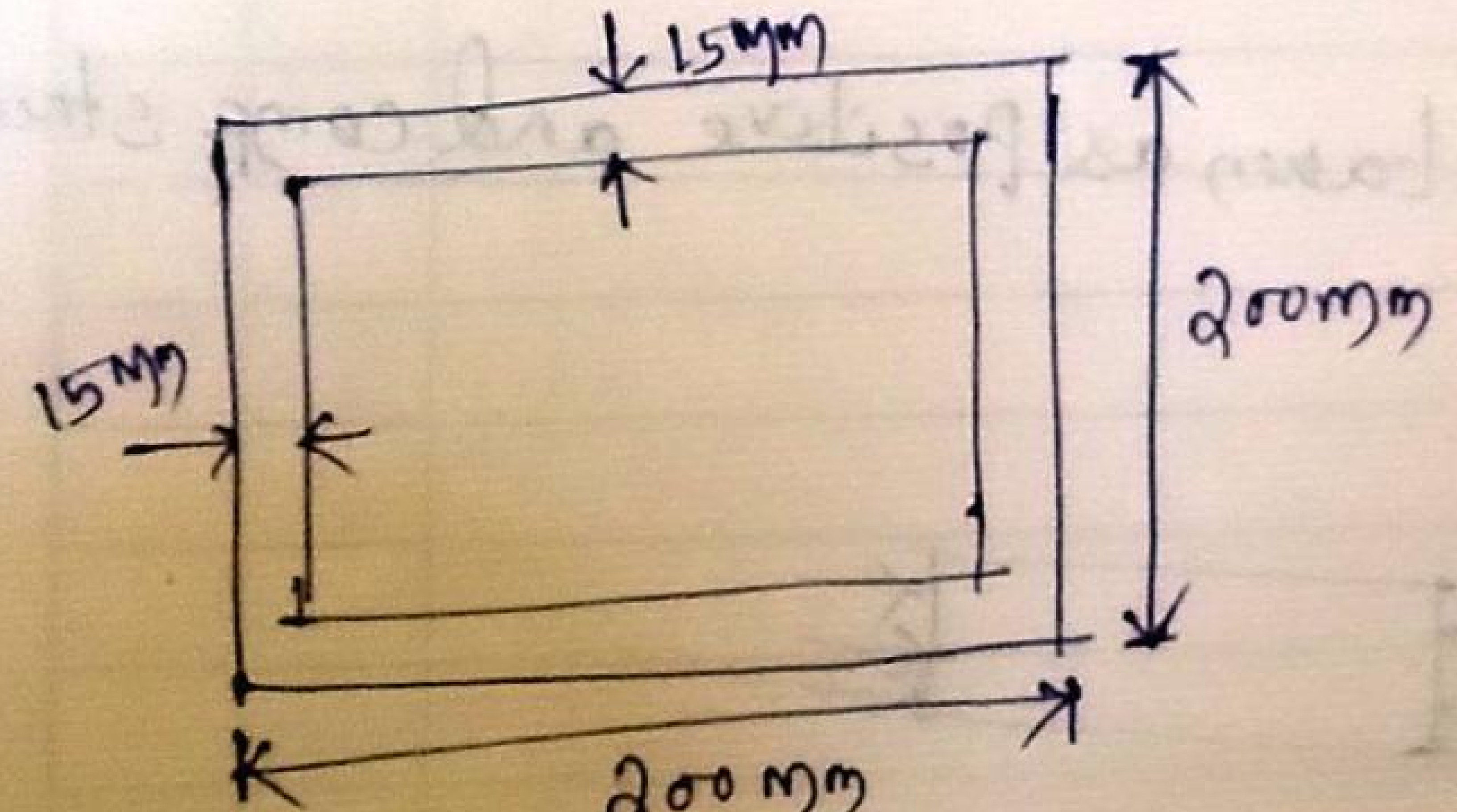
Q A two storey column ABC in a building is constructed with a hollow square box section below. The roof load at the top of column $P_1 = 80\,KN$ and the floor load at mid height is $P_2 = 100\,KN$. obtain the comp. stresses σ_{AB} and σ_{BC} at two section x−x and y−y respectively.



Area of c/s of column

$$A = \left[(200\times200) - (200-30)\times(200-30)\right]$$

$$= 40000 - 28900 = 11100\,mm^2$$



In portion AB, at section x−x column is subjected to axial force P_1.

So stresses will be $\sigma_{AB} = P_1/A = \dfrac{80\times10^3}{11100}$

$= 7.21\,MPa$.

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In portion BC, at section Y-Y column is subjected to

Total axial force. $P = P_1 + P_2$

$= 80 + 100 = 180 \, KN$.

So stresses will be

$$\sigma_{BC} = P/A = \frac{180 \times 10^3}{11100} = 16.22 \, N/mm^2.$$

Shear stress

It is also known as tangential stress.

Shear stresses are resistance offered by material against shearing force.

Its value is determined by dividing the shear force in the plane of the section by corresponding area.

Its unit is N/mm^2.

$$\boxed{\tau = S/A}$$

Shear stress may be of following two types.

(a) Direct shear stress :-

Direct shear stresses are produced due to direct shear force acting on the surface.

(b) Torsional stresses :- These stresses are produced when member is subjected to torsional moment or torque.

Note:- A shear stress in a given direction can't exist without a balancing shear stress of equal intensity in a direction at right angle to it.

This stress is called complementary shear stress.

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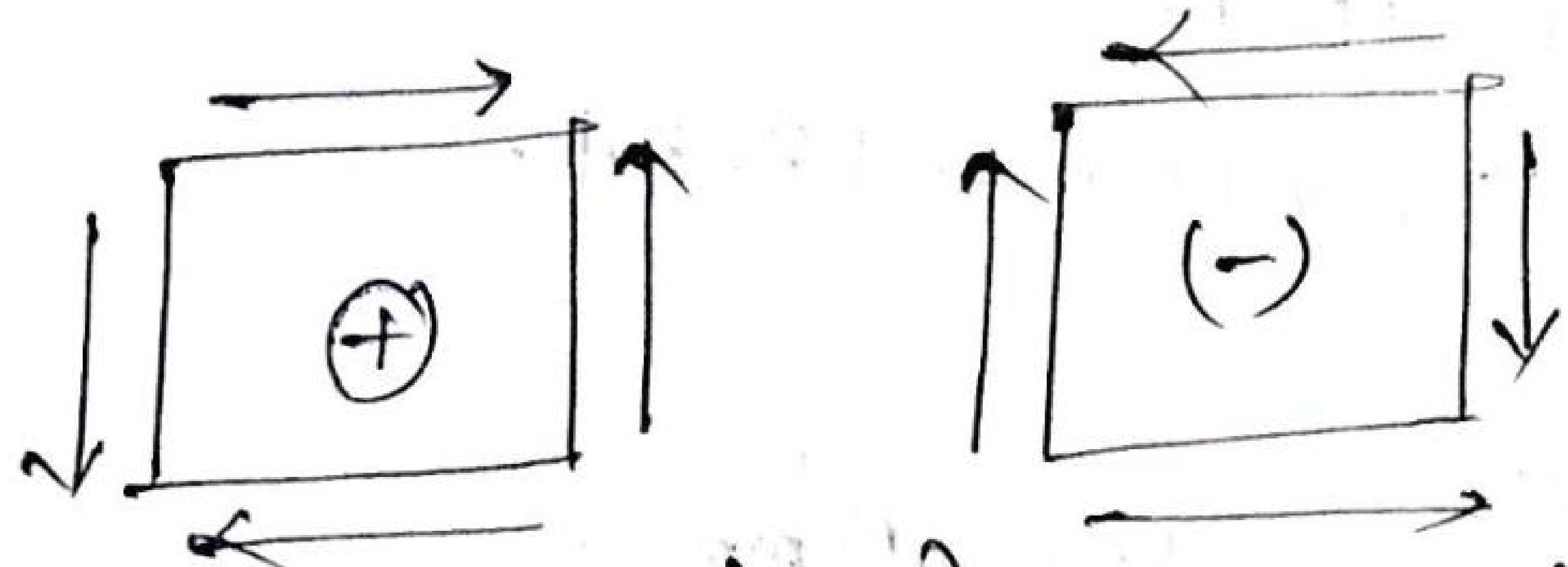
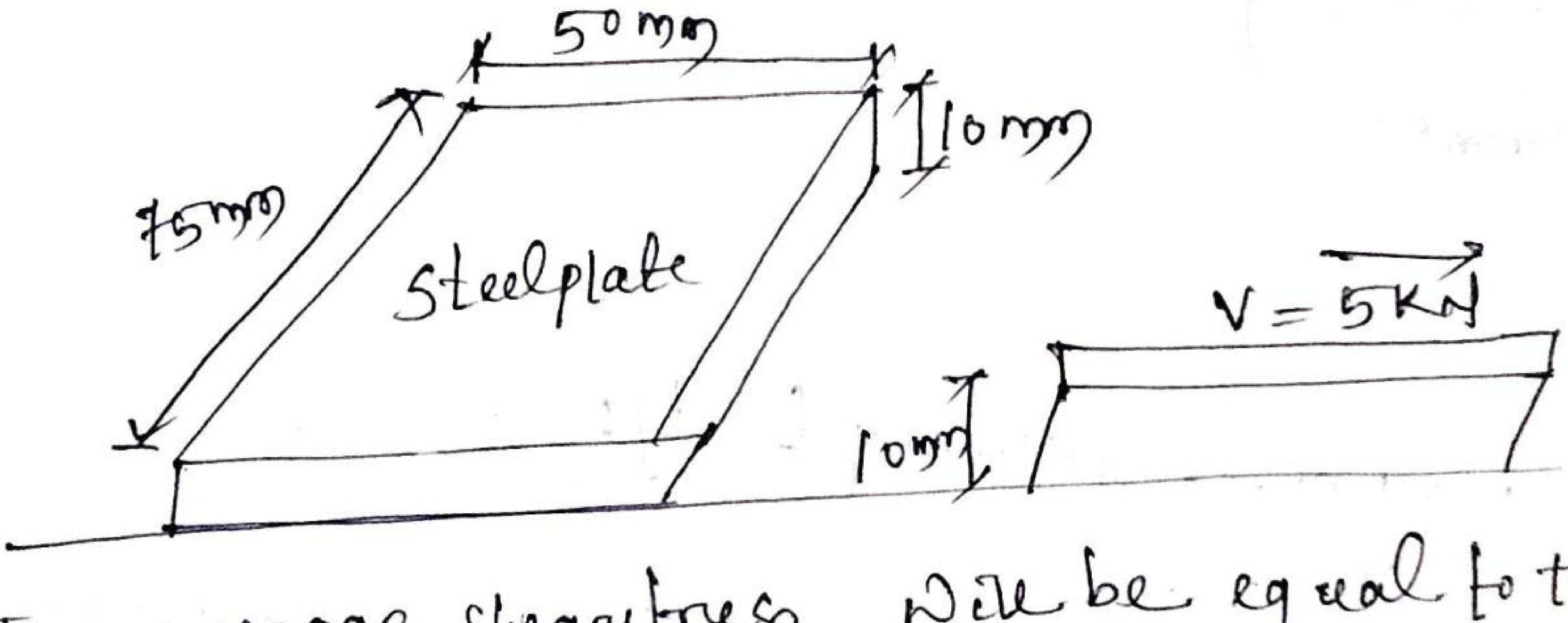
Sign convention



(postive shear stress) (Negative shear stress)

Q A bearing member consisting of a flexible material of thickness 10mm capped by a thin steel plate of dimension 50×75 mm is subjected to a horizontal S.F of 5KN. Determine the average shear stress.



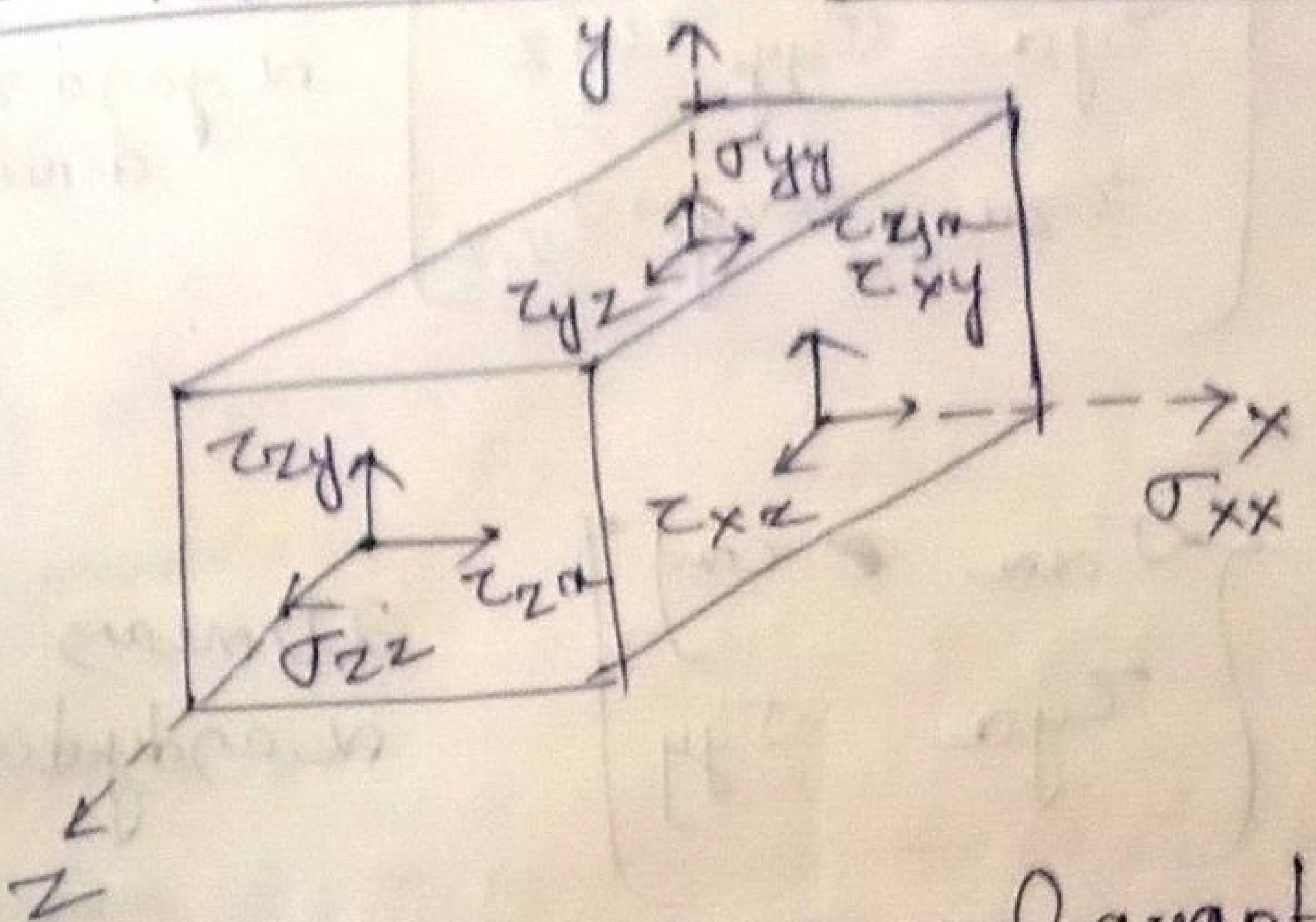
The average shear stress will be equal to the S.F divided by the area over which it acts.

$$\tau_{avg} = \frac{S}{A} = \frac{5\times10^3}{50\times75} = 1.33\ N/mm^2.$$

Ans.

Lecture- 3

Matrix Representation of stress and strain:-



Stress and strain are special quantities which are kept in a special group called Tensor.

" Tensor is defined by plane and plane is defined by two direction

→ In a 3D Loaded body at a point three mutually perpendicular plane exist.

→ At each plane there are 3 stress component. out of 3 one is Normal and other two are shear stress component

The stress in Normal plane is k/w as Normal stress (σ_{xx}) (σ_{yy} & σ_{zz})

and stresses which are in shear plane are k/w as shear stress [τ_{xy}, τ_{xz}, ~~τ_{yy}~~ τ_{yz}, τ_{yx} & τ_{zy}, τ_{zx}]

So including all 3 planes there can be nine stress components in which 3 stresses are normal and 6 are tangential / shear components.

→ Normal stresses are represented by σ_{xx}, σ_{yy}, σ_{zz} in which first letter represent plane and 2nd letter represents direction of stress.

3D Stress Matrix :-

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

3D means x, y and z direction

2D Stress Matrix :-

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{bmatrix}$$

2D means x and y direction

Strain :- Strain is defined as change in length to original length.

Types of strain :-

Strain are of following types

(1) Axial strain (ϵ)

(2) Lateral strain (ϵ_L)

(3) Shear strain. (ϕ)

Axial strain :- Strain in the direction of applied force is k/w as axial strain. or (linear strain) / Longitudinal

It is the ratio of change in linear dimension to original linear dimension.

$$\text{Axial strain} = \frac{\text{change in linear dimension}}{\text{original linear dimension}} = \frac{\Delta L}{L}$$

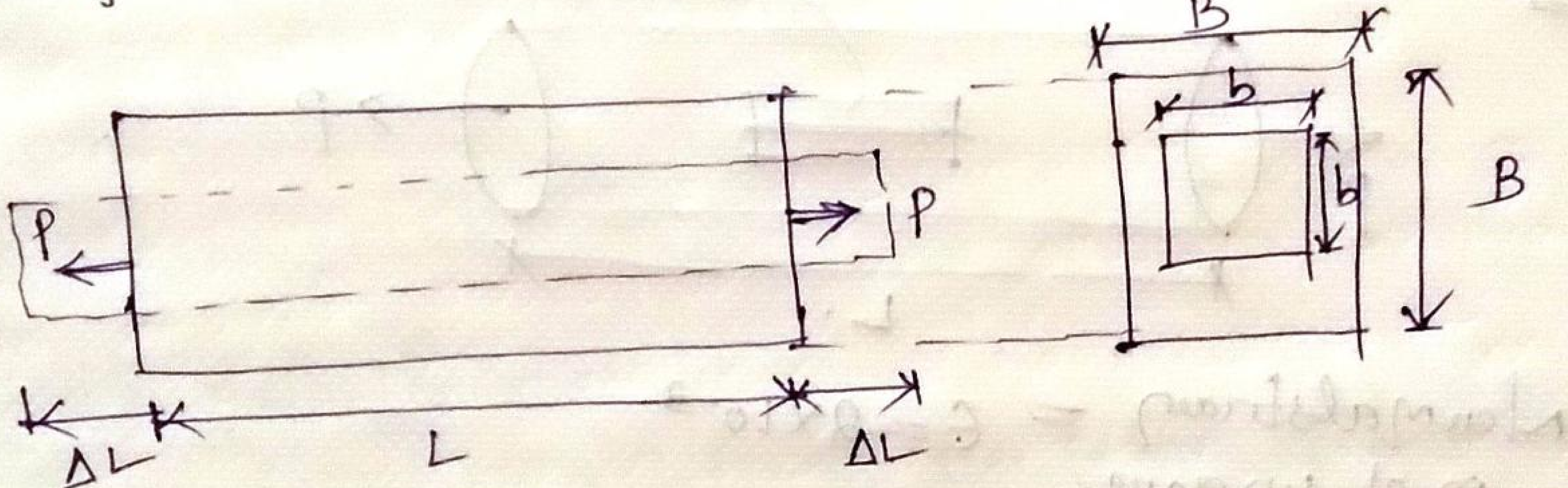
(ii) Lateral strain:

Strain in the perpendicular direction to the direction of applied force is k/was Lateral strain.

→ It is the ratio of change in Lateral dimension to original Lateral dimension.

$$\left[\text{Lateral strain } (\epsilon_L) = \frac{\text{change in lateral dimension}}{\text{original lateral dimension}} \right]$$

Consider a copper rod as shown in figure. 'L' is the Length and (B×B) is crossection which is pulled resty an axial force 'p'. The length of rod increases due to applied force but width and depth decreases.

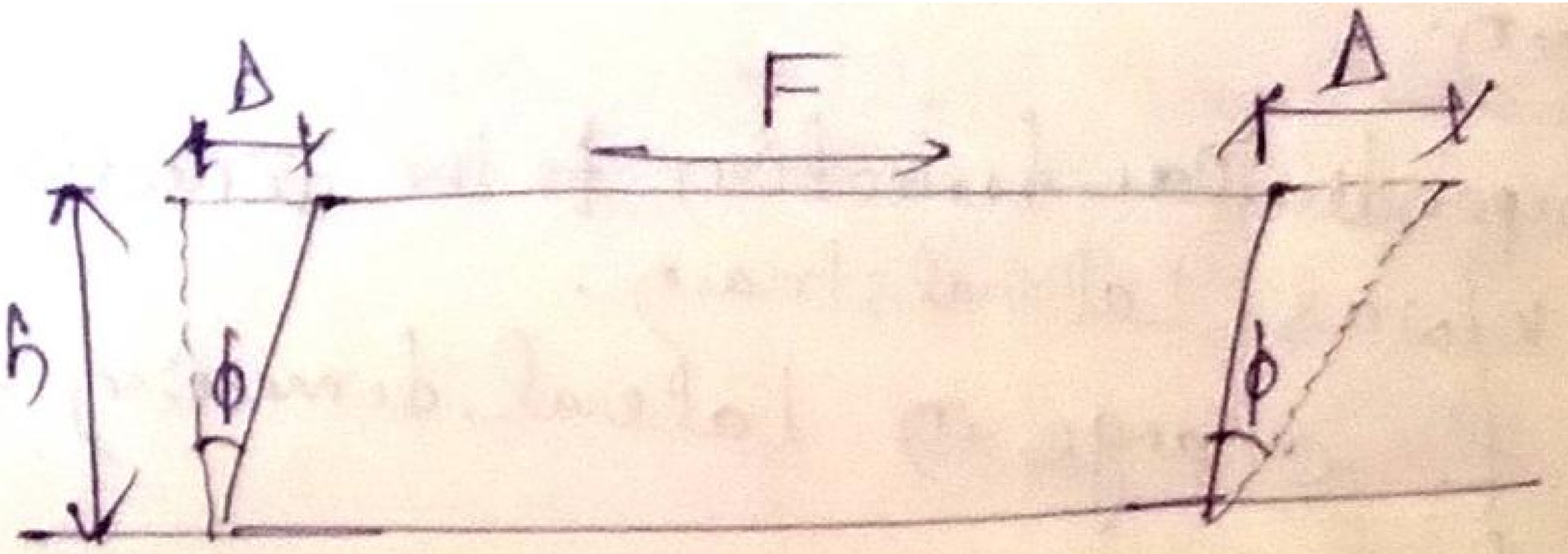


$$\text{Lateral strain} = \frac{\text{change in lateral dimension } (\Delta B)}{\text{original lateral dimension } (B)}$$

$$= \frac{\Delta B}{B} = \frac{(B-b)}{B}.$$

Shear strain:- Shear strains are angular deformation caused by shearing force.

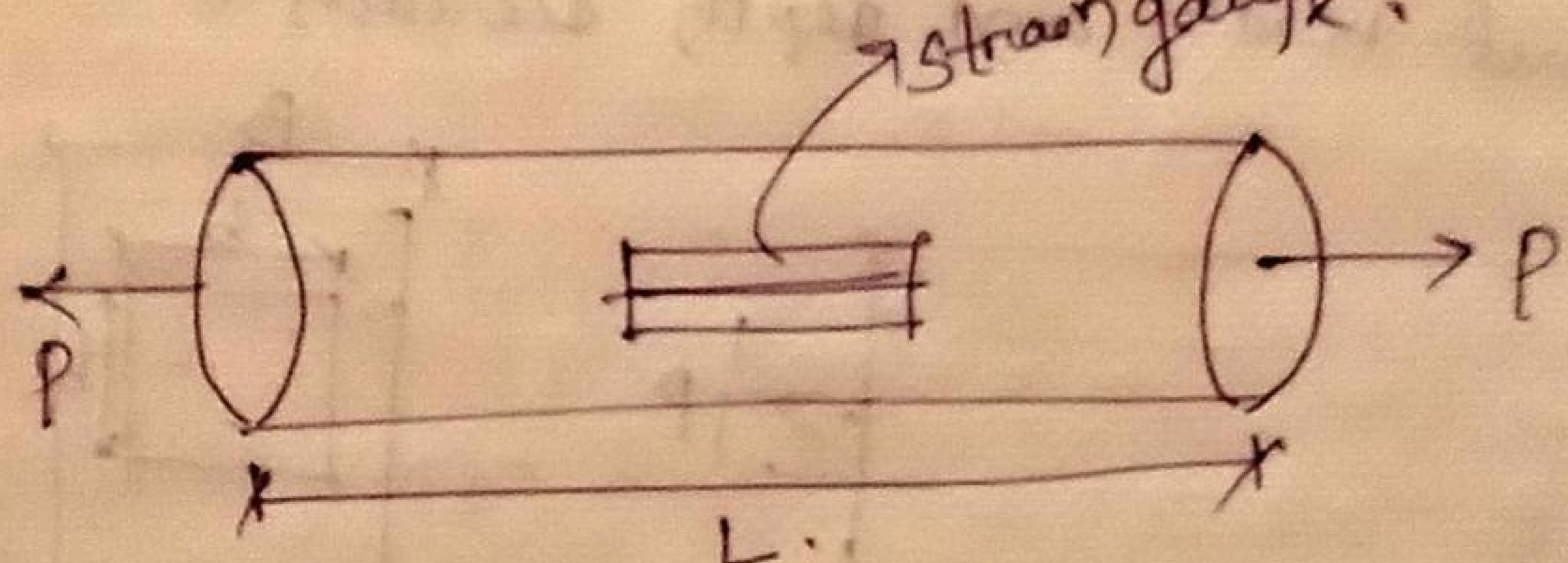
These are represented by ϕ.



Shearstrain $\boxed{\phi = \Delta/h}$

Q. A round bar of length $L = 1.5m$ is loaded in tension as shown in figure. A normal strain $\epsilon = 2\times10^{-3}$ is measured by a strain gauge placed on the bar. What elongation of the entire bar can be expected at this load?

Ans:



Normalstrain on strain gauge $= \epsilon = 2\times10^{-3}$.

Length of bar $(L) = 1.5m = 1.5\times1000mm$

We k/w linear / Longitudinal strain $\boxed{\frac{\Delta L}{L} = \epsilon}$

elongation $\Delta L = L \times \epsilon$

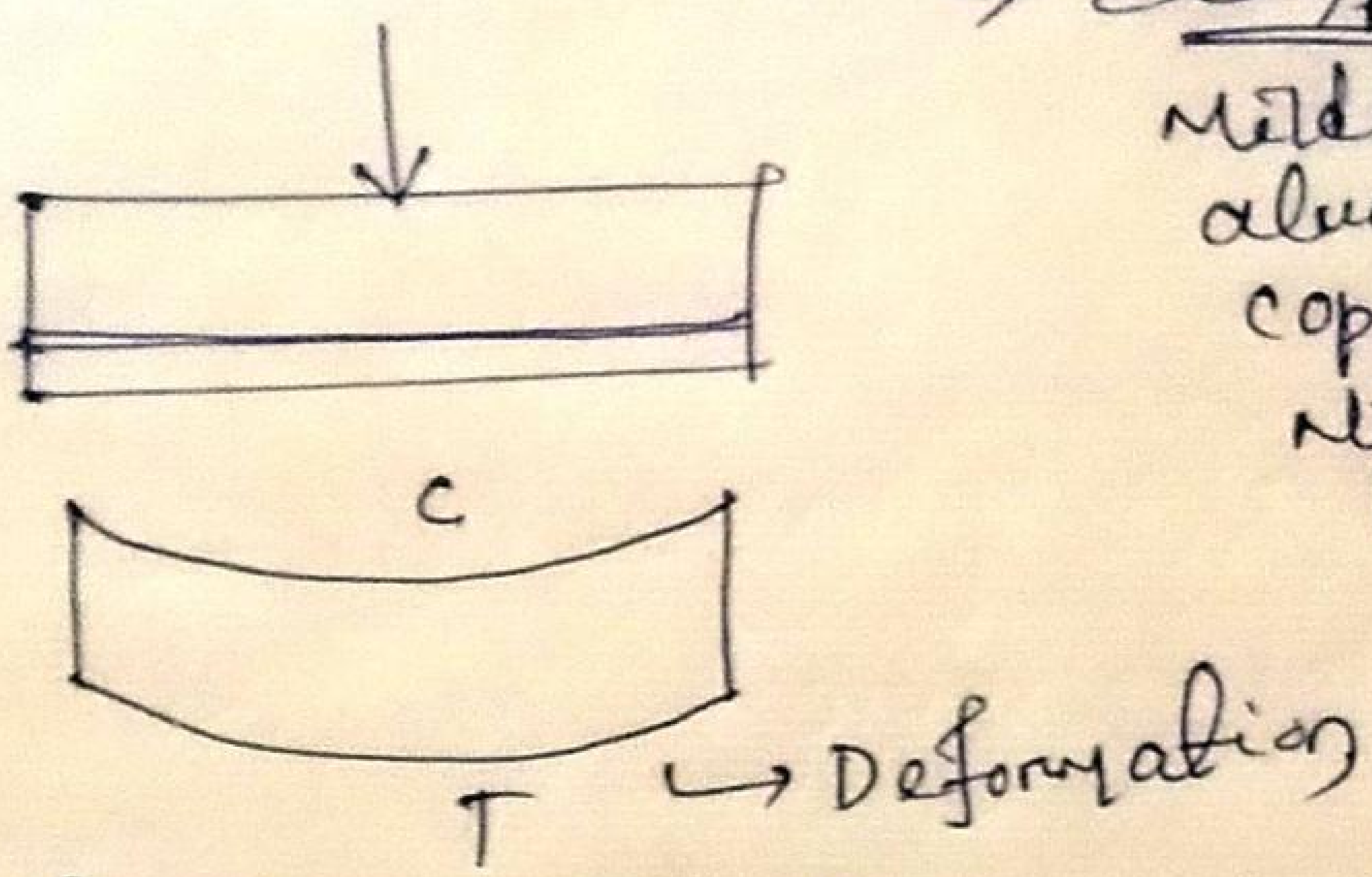
$= 2\times10^{-3} \times 1.5\times1000 = 3mm.$

Ductile Material:

Ductility is the property of the ductile material by which material can be stretched.

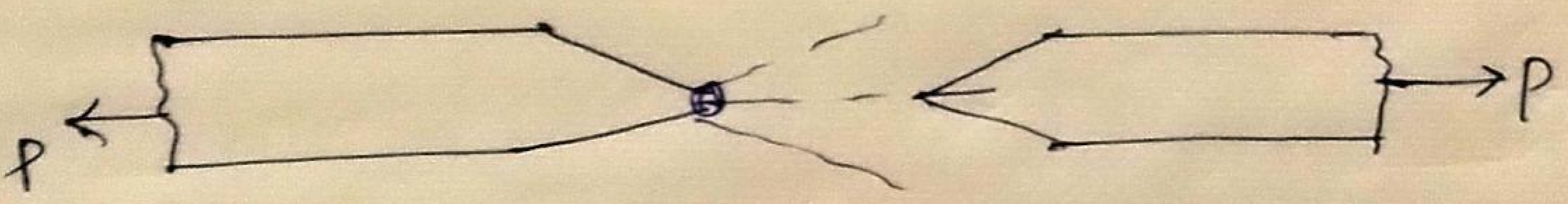
→ Large deformation are thus possible in ductile Material before rupture takes place.

→ Example
Mild steel
aluminium
copper.
Nickel
Brass.



Failure of ductile Material

ductile Materials are weak in shear and failure is due to shear strain forming 450 angle to the axis of specimen
→ cup and cone failure takes place in ductile metals.



Brittle Material :-

Brittleness happens in a Material due to lack of ductility.
Materials can't be stretched.
Materials fracture takes place immediately
→ In ~~of~~ brittle Material after elastic limit with a relatively smaller deformation

ex:- cast iron
concrete
glass.

Hooke's Law:

It states that under direct loading, within proportional limit stress is directly proportional to strain.

stress ∝ strain.

$\sigma \propto e$

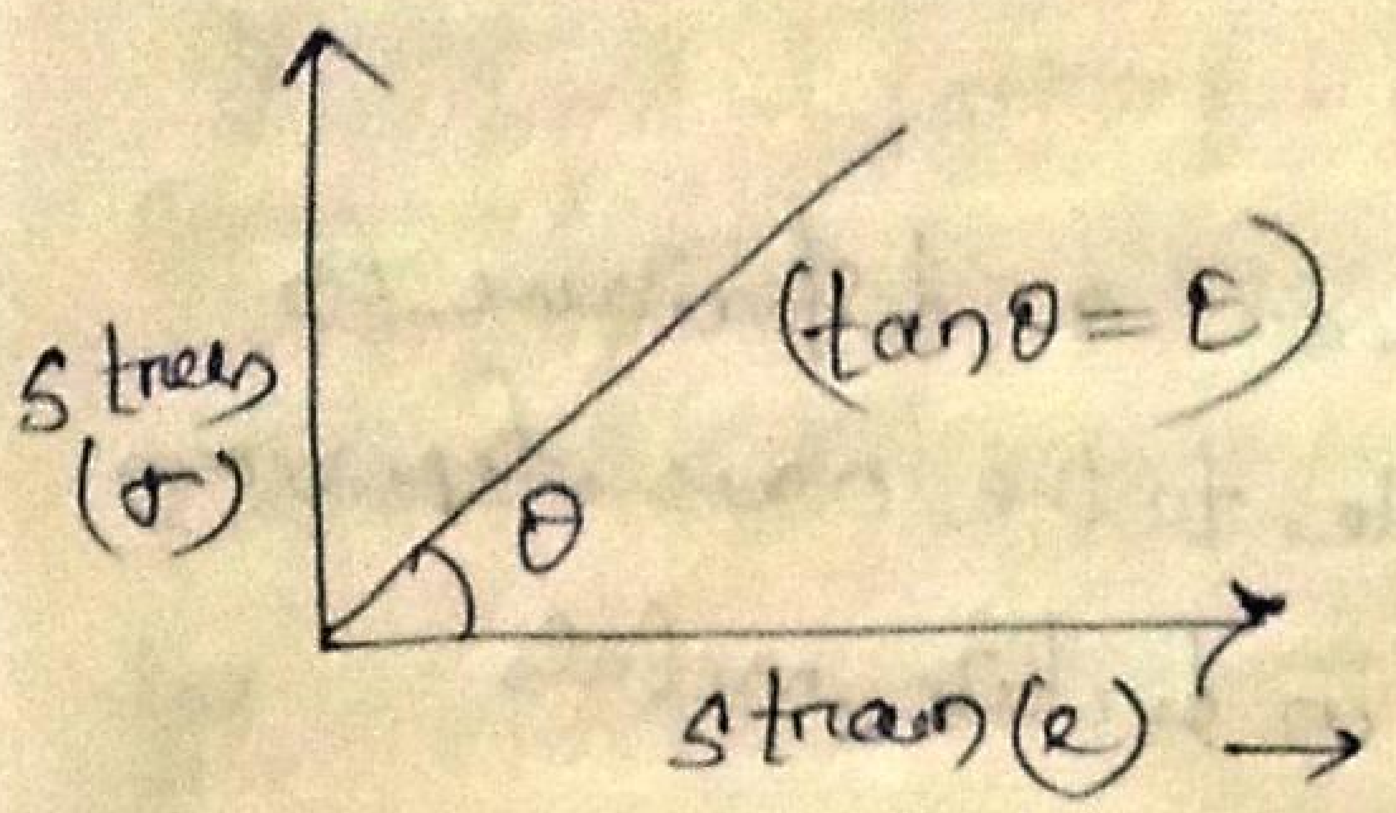
$\boxed{\sigma = Ee}$

$\rightarrow \boxed{E = \sigma/e}$

E = Modulus of elasticity.

σ = stress

e = strain.



where E is constant of proportionality and known as young's Modulus of elasticity (slope of stress-strain curve)

* Hooke's Law is valid up to the limit of proportionality. However for Mild steel proportionality limit and elastic limit are almost equal.

But for other metals elastic limit may be higher than proportionality limit. (eg - Rubber).

→ The slope of stress-strain curve is called Modulus of elasticity (E).

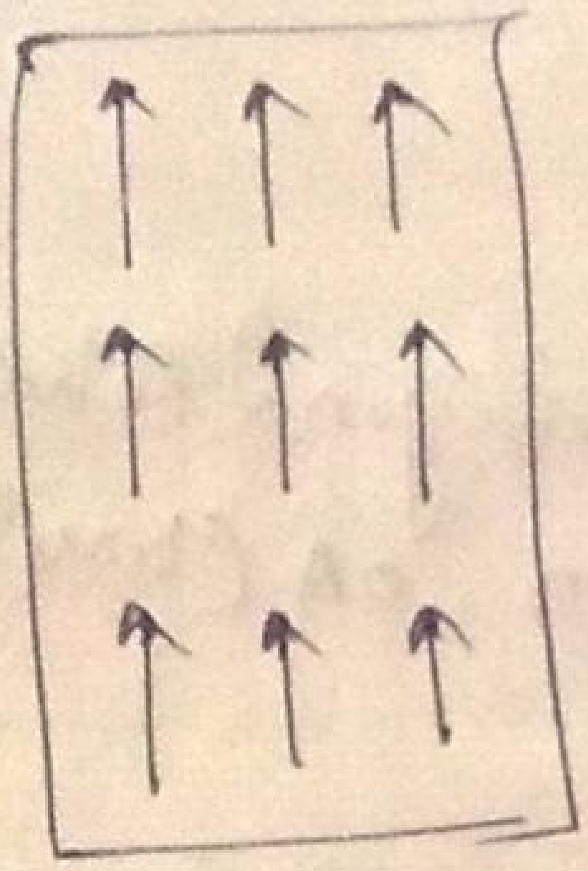
→ (The Modulus of elasticity (E) is the constant of proportionality which is defined as the intensity of stress that causes unit strain.

→ unit of Modulus of elasticity :- N/mm2

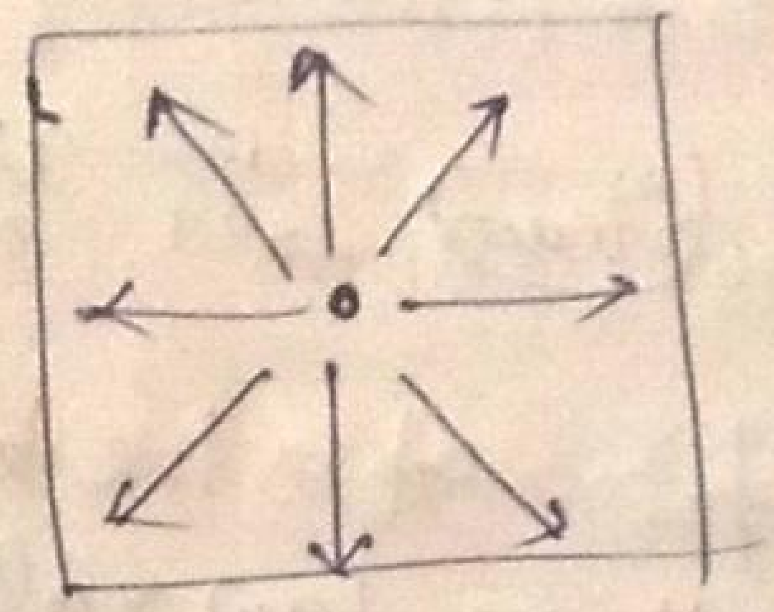
same as stress.

Assumption of Hooke's law:

(1) Material is homogenous (properties are equal at all points)
(2) Material is Isotropic (properties are equal in all direction)
(3) Material is elastic.



[Homogenous]



[Isotropic]

Note:-

→ If properties of Material are equal in all direction called Isotropic Material.

→ If properties are different in all direction then Material are called as Non-isotropic Material.

and if properties are different in three Mutually ⊥r direction then Material is called as orthotropic Material.
eg-crystal.

* Tension test for Mildsteel:-

The Mechanical properties of Materials used in ~~eg~~ engg. are determined by experiments on small specimen.

As steel is strong in tension and weak in compression.

Stress-strain curve for Tension:

Tension test for mildsteel performed on UTM (universal testing Machine)

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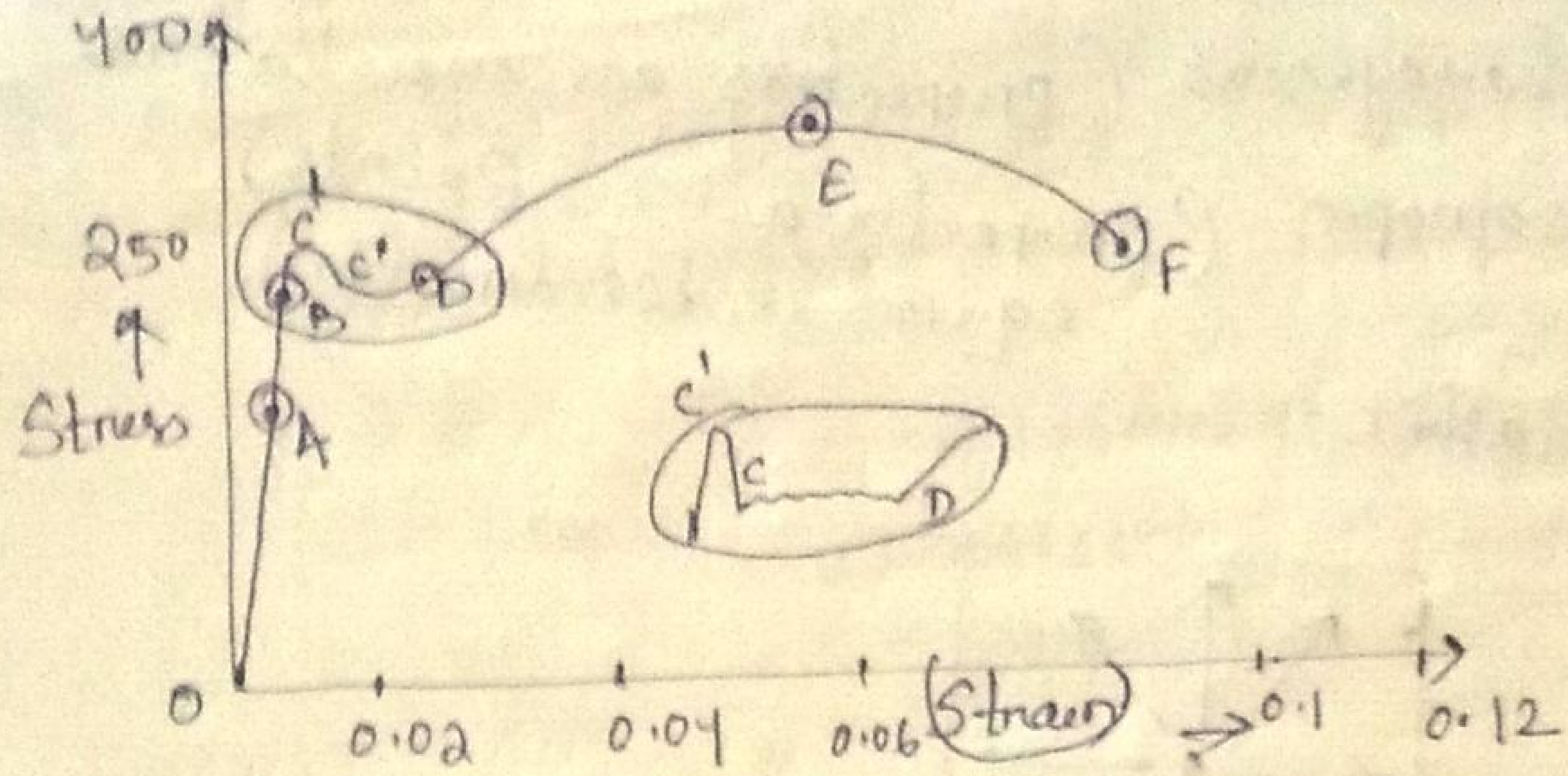


A is limit of proportionality :- It is the limit beyond which linear variation ceases. Hooks Law valid in OA (Limit of proportionality).

B is elastic limit :- The Maximum stress upto which a specimen regains its original length on removal of application of load. means it is elastic limit

→ for mild steel 'B' is very nearer to A but for other material B may be greater than A.

C' upper yield point :- The magnitude of stress corresponding to C' depends on the C/S area, shape of specimen and the type of equipment used for test.

C Lower yield point :- This is also called as actual yield pt. (Limit of elastic behavior and beginning of plastic behavior).

→ The stress at 'C' is yield stress with value ($\sigma_y = 250$ N/mm²)

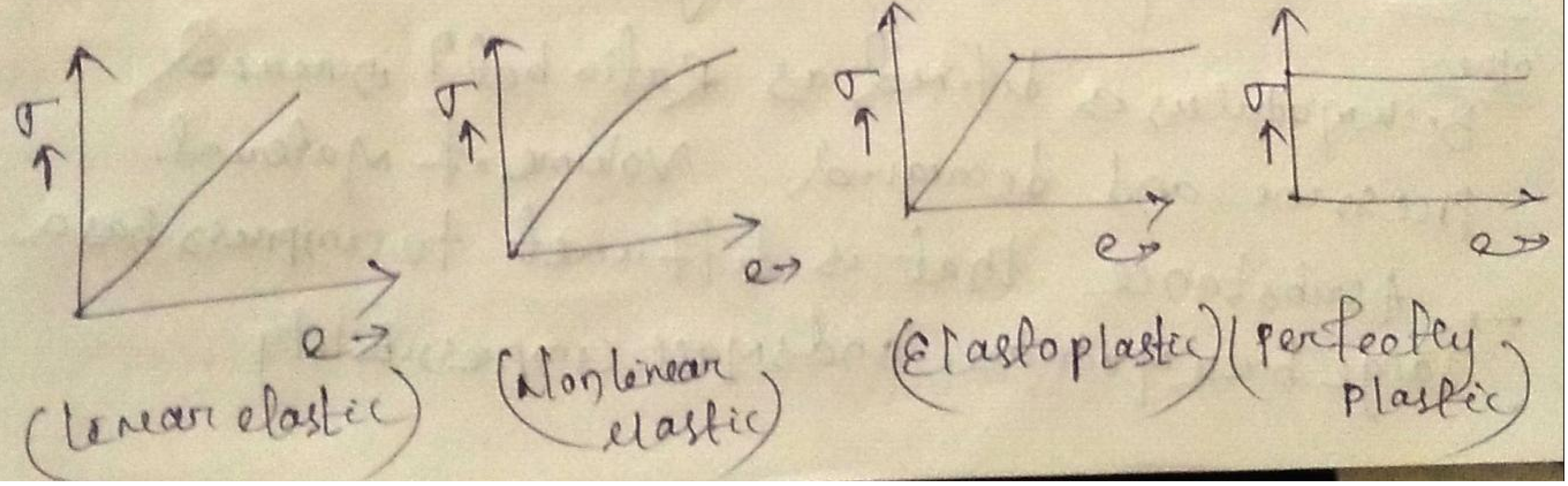
CD :- It represents perfectly plastic region.
It is the strain which occurs after the yielding point 'C' without any increase in stress.

DE :- It represent strain hardening.
In this region further addition of stresses give additional strain.
→ The strain increases in faster rate in this region.
→ The Material in this range undergoes further deformation
This portion is n't used for structural design.

E ultimate point :- The stress corresponding to this point is ultimate stress : (Maximum stress that a material can withstand). (It is 20% of mild steel)
F :- It is fracture point :- Stress corresponding to this is called as Breaking stress. (It is 25% of mild steel)
→ The strain is k/w as fracture strain.
* Region between E and F is necking region in which area of c/s is drastically decreased.

Note
Strain that occours before the yield point is k/w as elastic strain and which occours after yield point is called plastic strain.
→ for mild steel plastic strain is 10 to 15 times of elastic strain.

Types of Material behavior



Elastic constants :-

(1) Young's Modulus :- (E) It is the slope of stress-strain curve under direct loading.

from hooke's Law

$\sigma \propto e$

$\boxed{E = \sigma/e}$ $\boxed{E = \frac{\text{Direct stress}}{\text{Direct strain}}}$

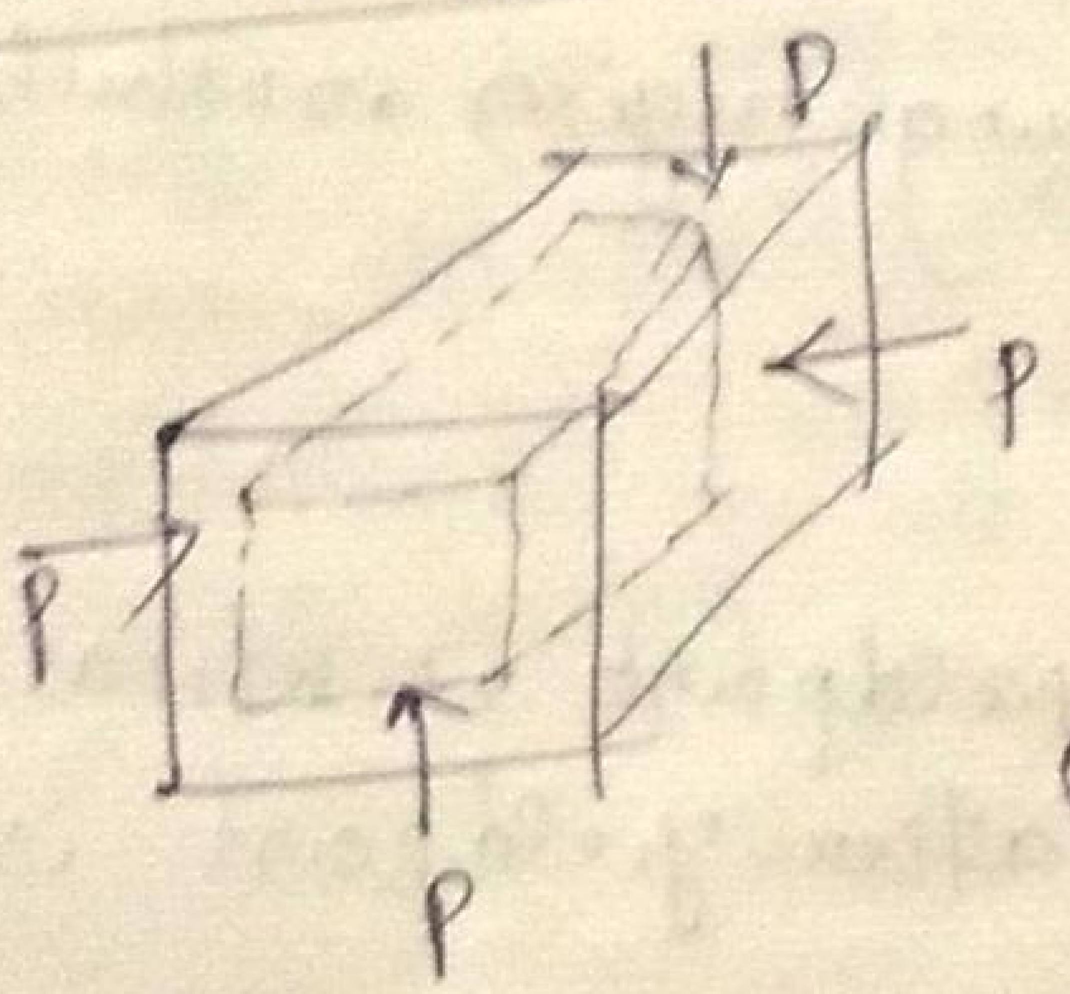
$\sigma = Ee$

$\boxed{E = \sigma/e}$

(2) Bulk Modulus :- (K)

The ratio between direct stress to volumetric strain is k/w as Bulk Modulus.

$$\boxed{K = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{\sigma}{e_V}}$$



Significance of bulk Modulus With respect to compressibility in 3D Loading

(*) Bulk Modulus is reciprocal of compressibility

$\epsilon_V = e_V = \Delta V/V$

Bulk Modulus $\boxed{K = \frac{\sigma}{e_V} = \frac{\sigma}{(\Delta V/V)}}$

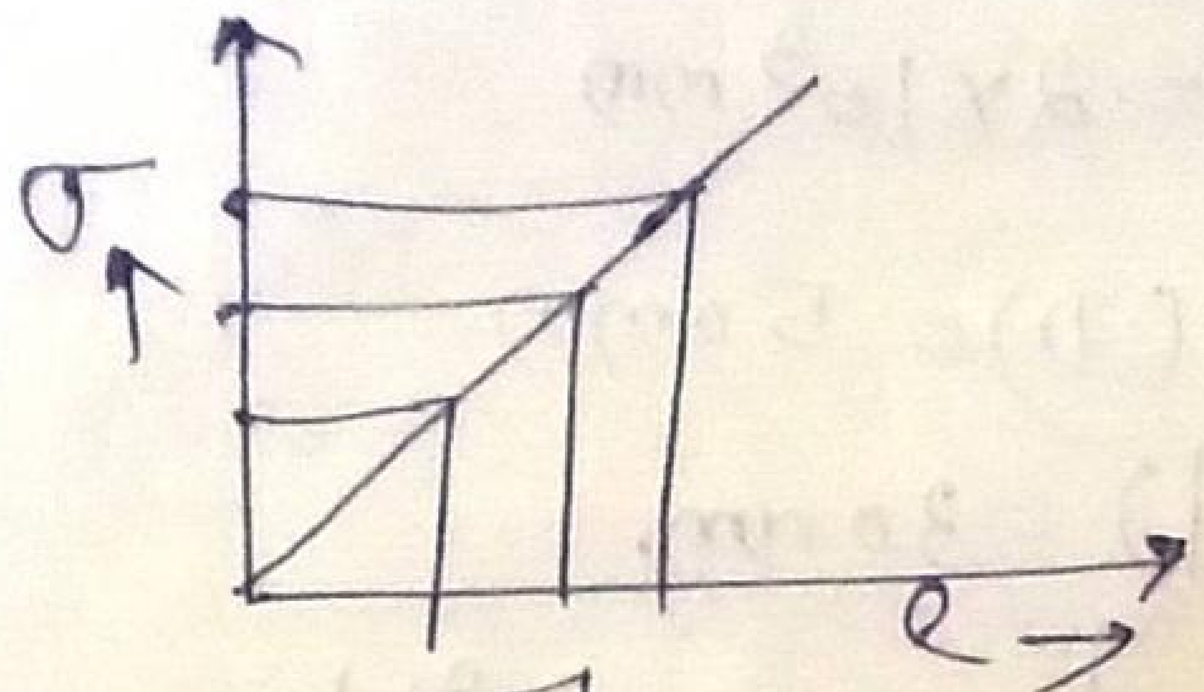
Note :- Bulk Modulus is inversely proportional to compressibility

other

Bulk modulus is defined as Ratio betn increased pressure and decreased Volume of Material.

→ A substance that is difficult to compress has a large bulk Modulus and small compressibility.

Withi in elastic limit stress $\propto$ strain Means



If stress increase then strain will also increase.

$\boxed{\sigma \propto e}$ ∴

*** Deformation of a Body due to force acting on it**

Consider a body subjected to a tensile stress.

P = Load or force acting on the body

L = Length of the body

A = c/s area of body

σ = stress induced on the body

E = Modulus of elasticity for the material

e = strain

δl = Deformation of the body.

$\sigma = P/A$, $e = \dfrac{\sigma}{E}$

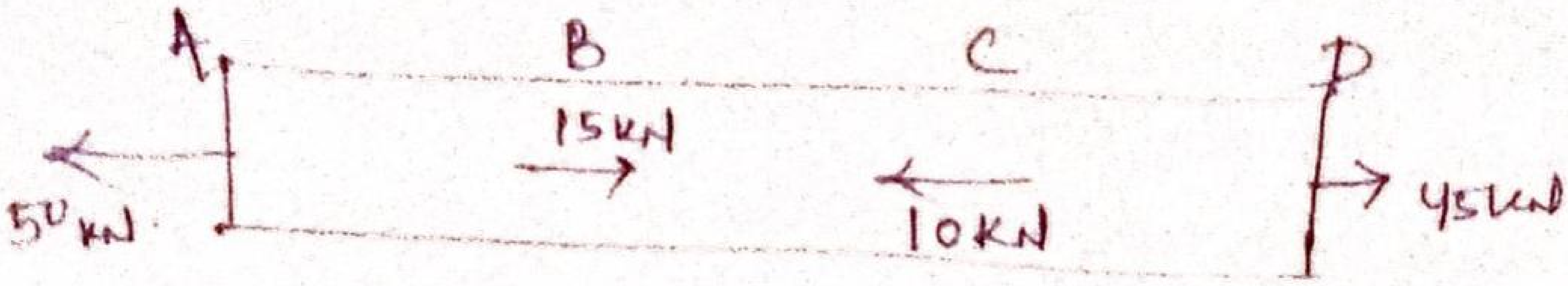
$$= \frac{P/A}{E} = \frac{P}{AE}$$

$$\cancel{\frac{\delta L}{L} = \frac{P}{AE}} \qquad \Rightarrow \frac{\delta l}{l} = \frac{P}{AE}$$

$$\frac{\delta L}{L} = e \qquad \boxed{\Rightarrow \delta L = \frac{PL}{AE}}$$

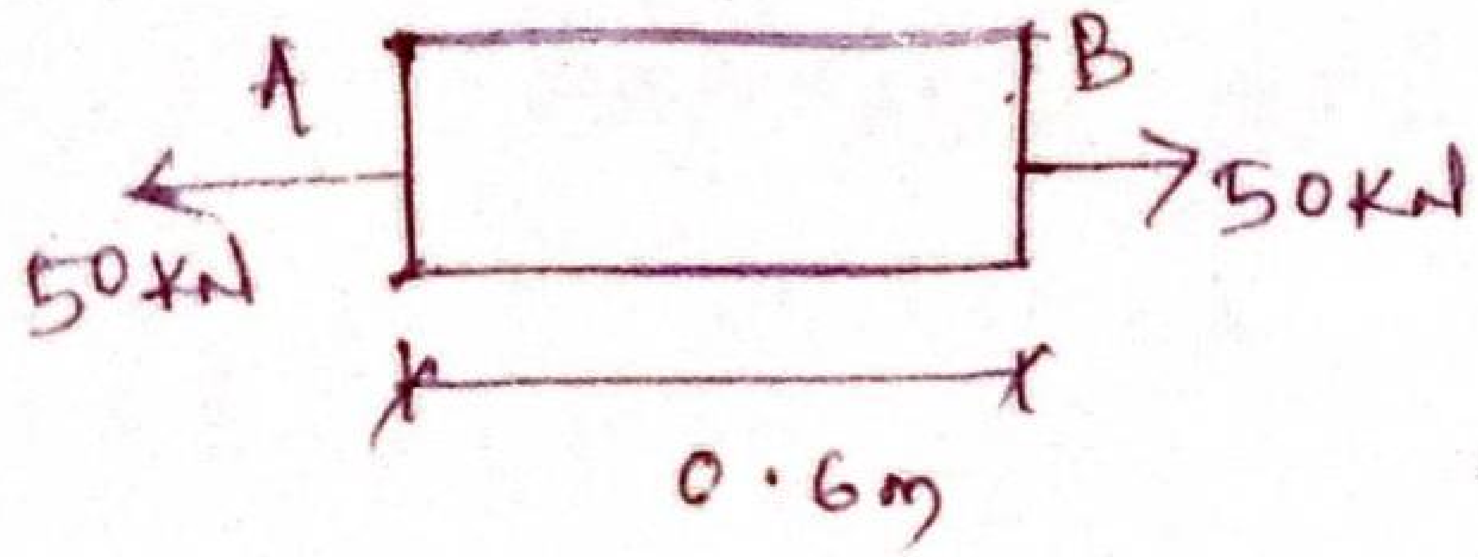
$$\delta L = e \times L$$

(2) A steel bar of cross section 500 mm^2 is acted upon by the forces shown in figure. Determine the total elongation of the bar. Consider $E = 2 \times 10^5\ N/mm^2$



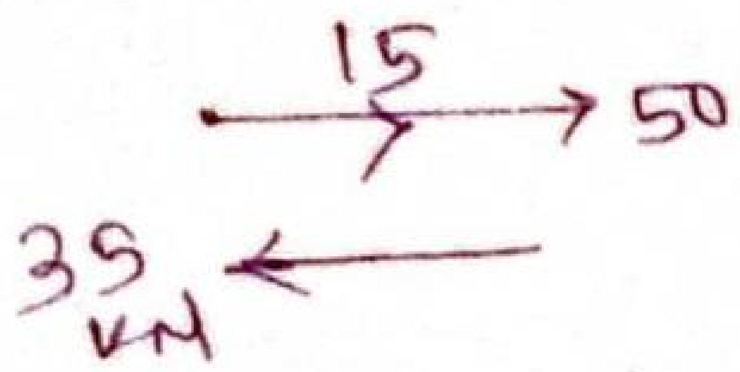
Solution: Total deflection/elongation (Δ) will be equal to sum of elongation of Δ_{AB}, Δ_{BC} and Δ_{CD} OR $\delta_{AB}, \delta_{BC}, \delta_{CD}$

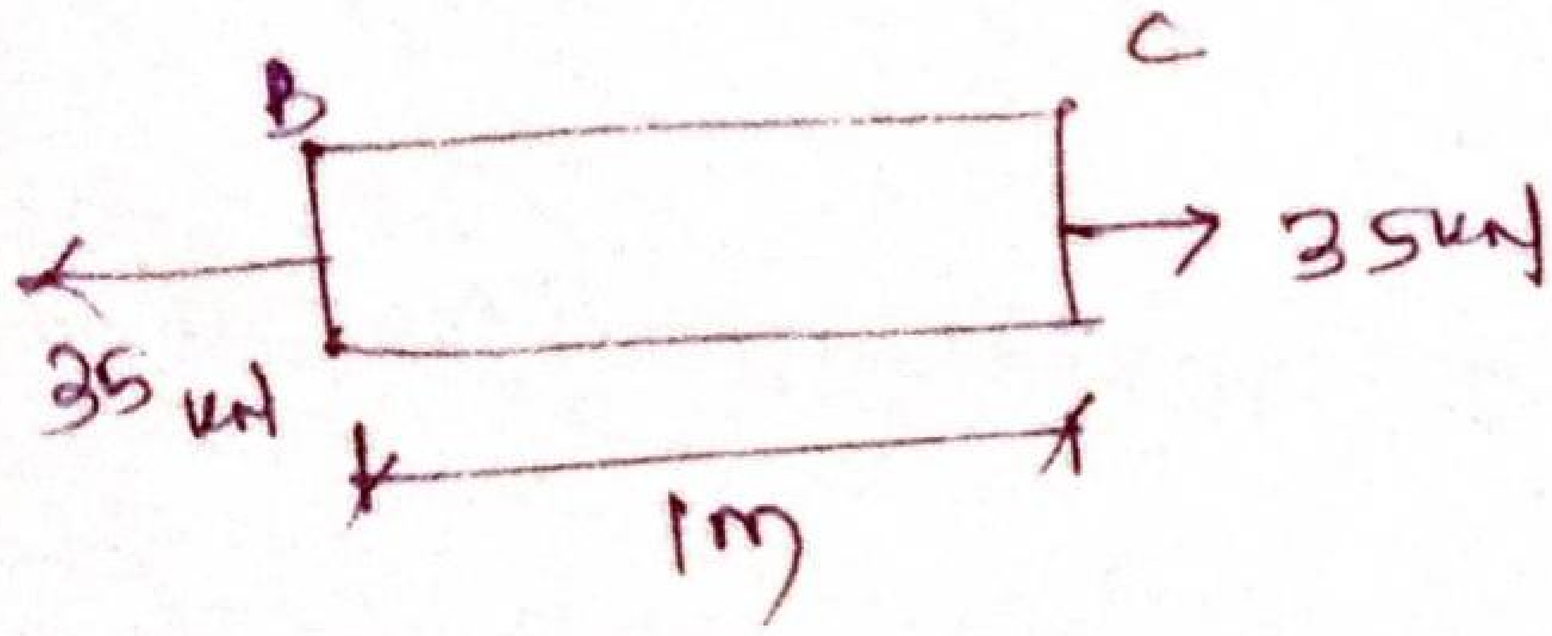
For portion AB



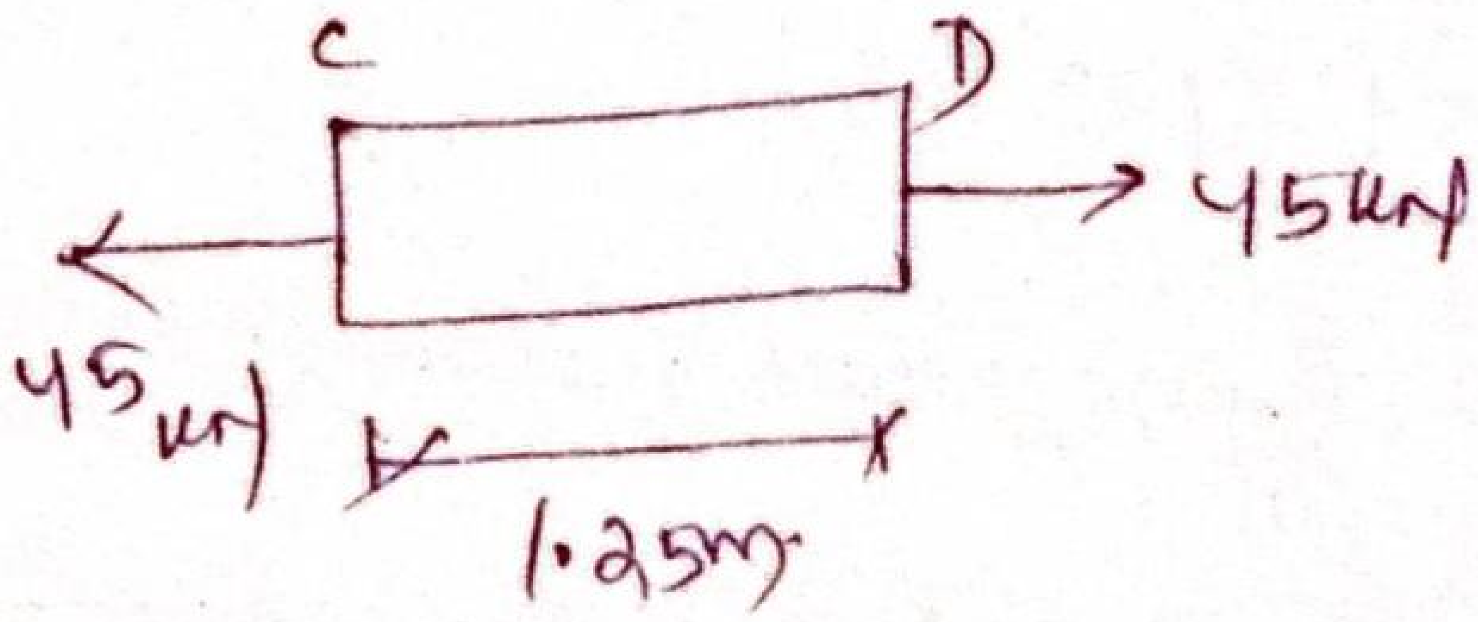
$$\left[\because \Delta = \frac{PL}{AE}\right]$$

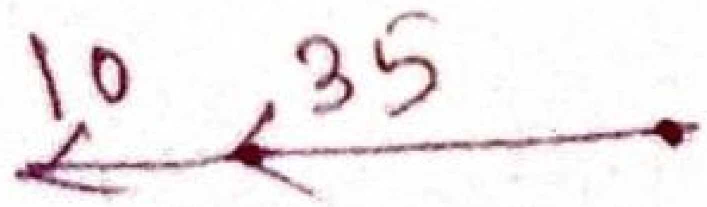
For portion BC





For portion CD





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$$\Delta_{AB} = \frac{PL}{AE}$$

$$= \frac{50\times10^3 \times 0.6\times1000}{500\times2\times10^5} = +0.3\,mm \quad \text{(Elongation)}$$

$$\Delta_{BC} = \frac{PL}{AE} = \frac{35\times10^3\times1000}{500\times2\times10^5} = +0.35\,mm \quad \text{(Elongation)}$$

$$\Delta_{CD} = \frac{PL}{AE} = \frac{45\times10^3\times1.25\times1000}{500\times2\times10^5} = +0.5625\,mm$$

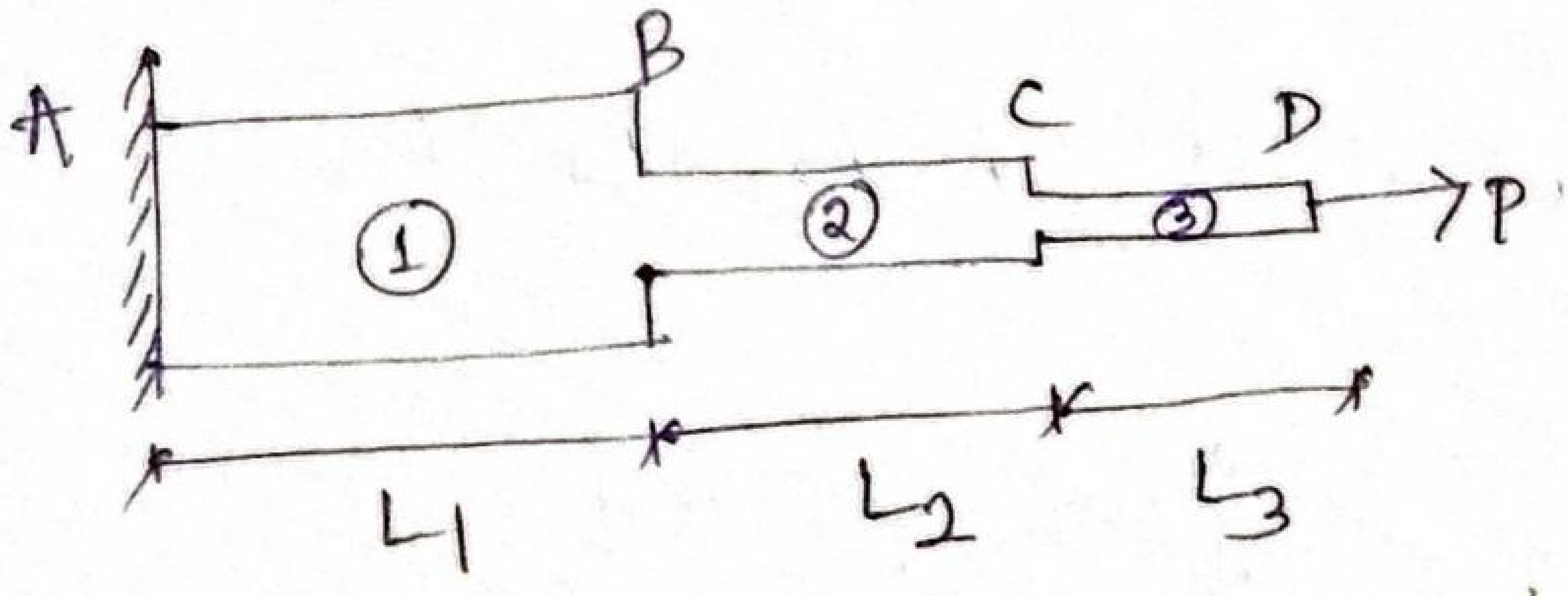
(Elongation)

Total elongation $= \Delta = \Delta_{AB} + \Delta_{BC} + \Delta_{CD}$

$$= 0.3 + 0.35 + 0.5625$$

$$= +1.2125\,mm \quad \text{(Tensile)}.$$

* Axial defection of varying cross-section Bar



Case-1:- If materials are same $\left[\Delta = \Delta_1 + \Delta_2 + \Delta_3\right]$

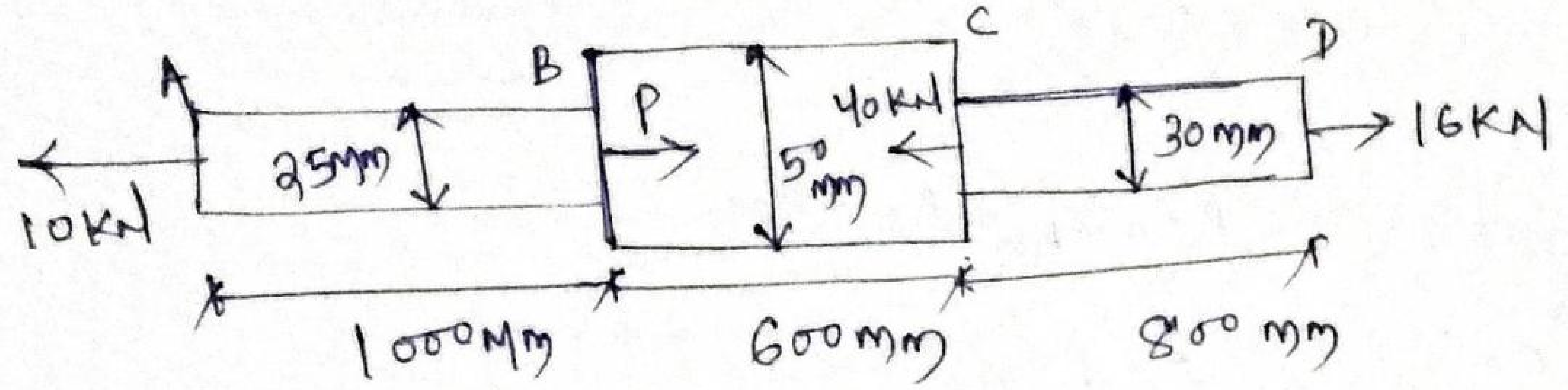
$$= \frac{PL_1}{A_1E} + \frac{PL_2}{A_2E} + \frac{PL_3}{A_3E}$$

$$\Delta = \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right]$$

Case-2 If materials are also different then

$$\Delta = P\left[\frac{L_1}{E_1A_1}+\frac{L_2}{E_2A_2}+\frac{L_3}{E_3A_3}\right]$$

Q A member ABCD is subjected to point loads as shown below. Calculate the force 'P' necessary for equilibrium. Taking modulus of elasticity as $2.05\times10^5\ N/mm^2$. Determine the total elongation of the member.



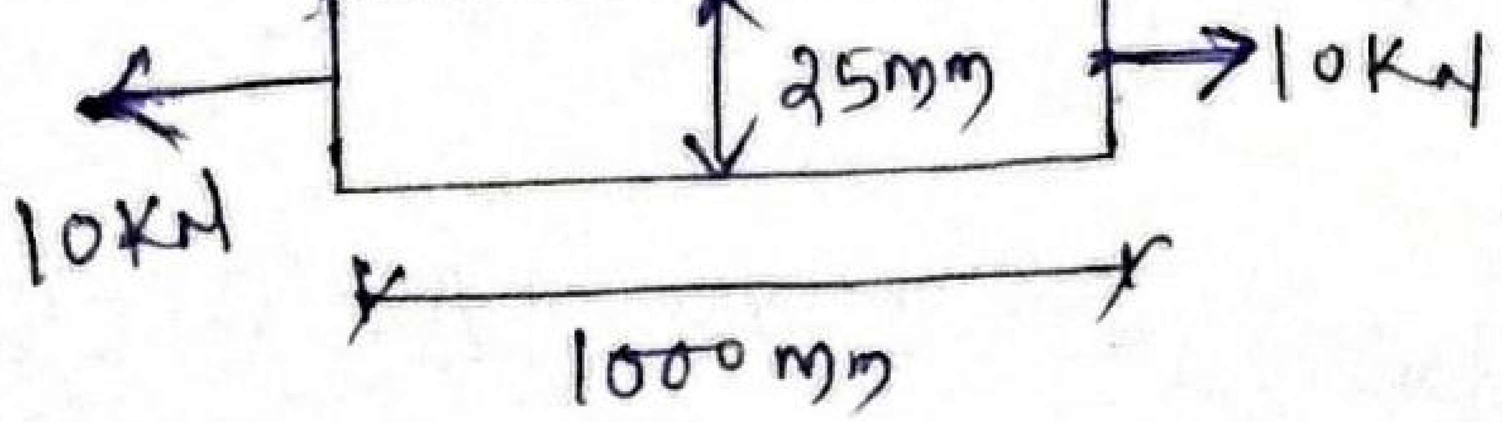
Solution for equilibrium left side forces = Right side forces

$10+40 = P+16$

$\Rightarrow P = 34\ KN\ (\rightarrow)$

Total elongation of bar will be $\Delta = \Delta_{AB}+\Delta_{BC}+\Delta_{CD}$.

for portion AB



$$A = \frac{\pi d^2}{4} = \frac{\pi\times25^2}{4} = 490.9\ mm^2$$

$$\Delta_{AB} = \frac{PL}{AE} = \frac{10\times10^3\times1000}{490.9\times2.05\times10^5} = +0.099\ mm$$

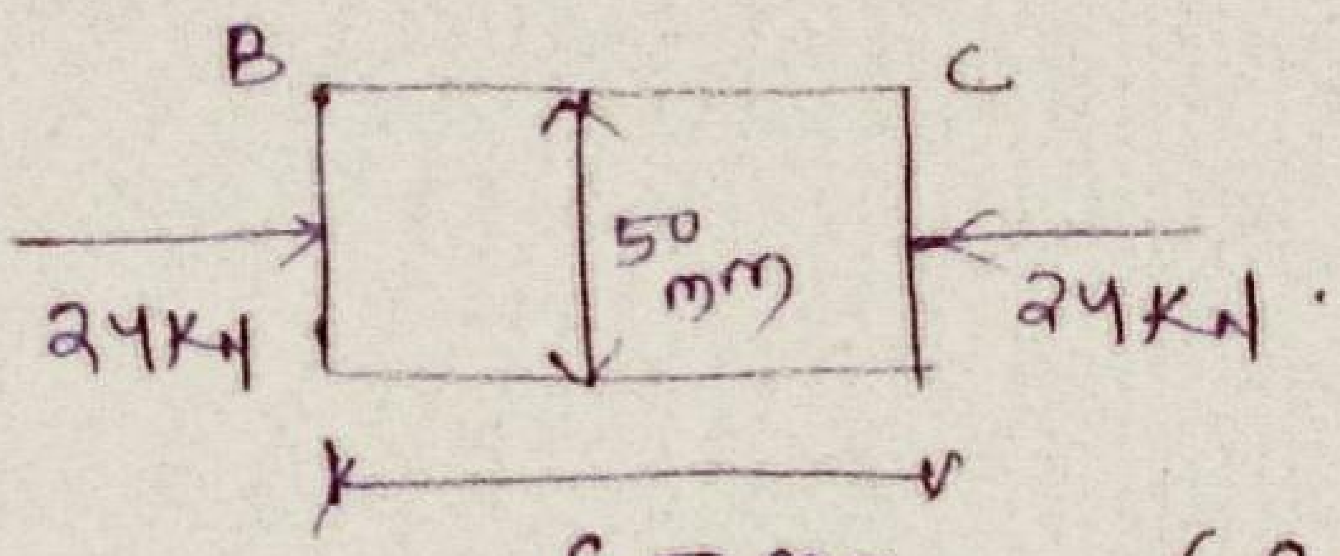
(Elongation)

for portion BC

$\xrightarrow{10}$

P = 34 KN.

$34 - 10 = 24\,KN$



(Contraction)

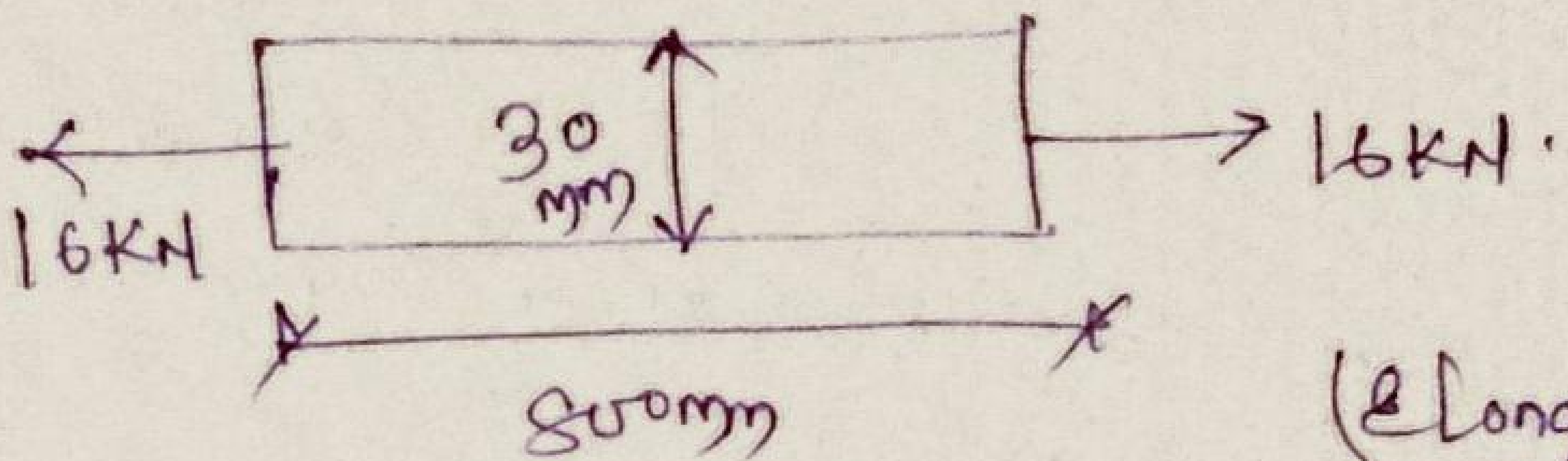
$A = \pi/4 \times d^2 = \pi/4 \times (50)^2 = 1963.5\,mm^2$

$$\delta_{BC} = \frac{PL}{AE} = \frac{24 \times 10^3 \times 600}{1963.5 \times 2 \times 10^5} = -0.036\,mm.$$

for portion CD

$40 \leftarrow 24 \leftarrow 0$

$40 - 24 = 16\,KN$



(elongation)

$A = \pi/4 \times (30)^2 = 706.9\,mm^2.$

$$\Delta_{CD} = \frac{PL}{AE} = \frac{16 \times 10^3 \times 800}{706.9 \times 2.05 \times 10^5} = +0.088\,mm.$$

Total elongation or contraction

$$\Delta = \Delta_{AB} + \Delta_{BC} + \Delta_{CD}$$

$$= +0.099 - 0.036 + 0.088 = +0.151\,mm$$

(elongation)

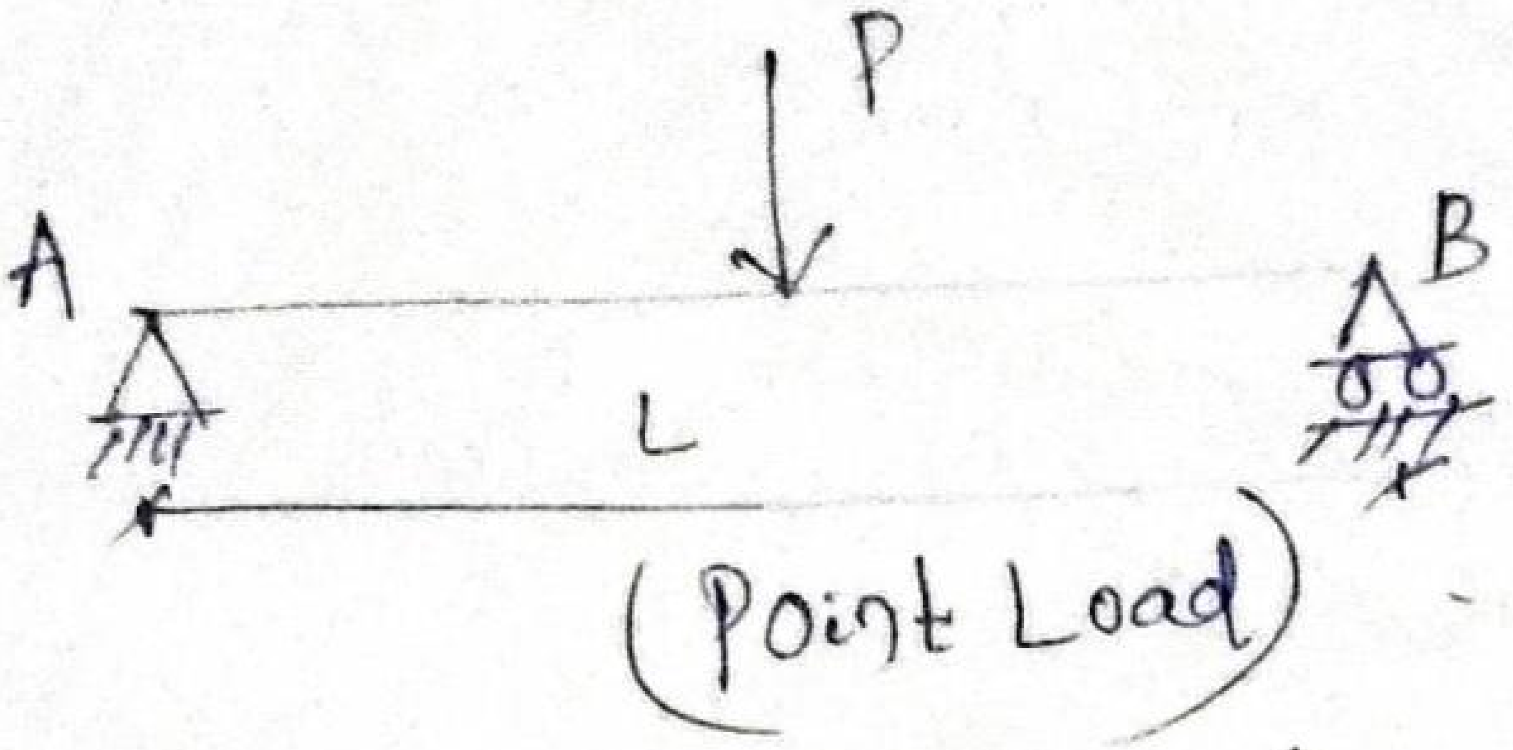
If answer will be -ve then contraction

Here answer is +ve so elongation or expansion of Member.

* Shearforce and Bending Moment :-

Types of Loading :

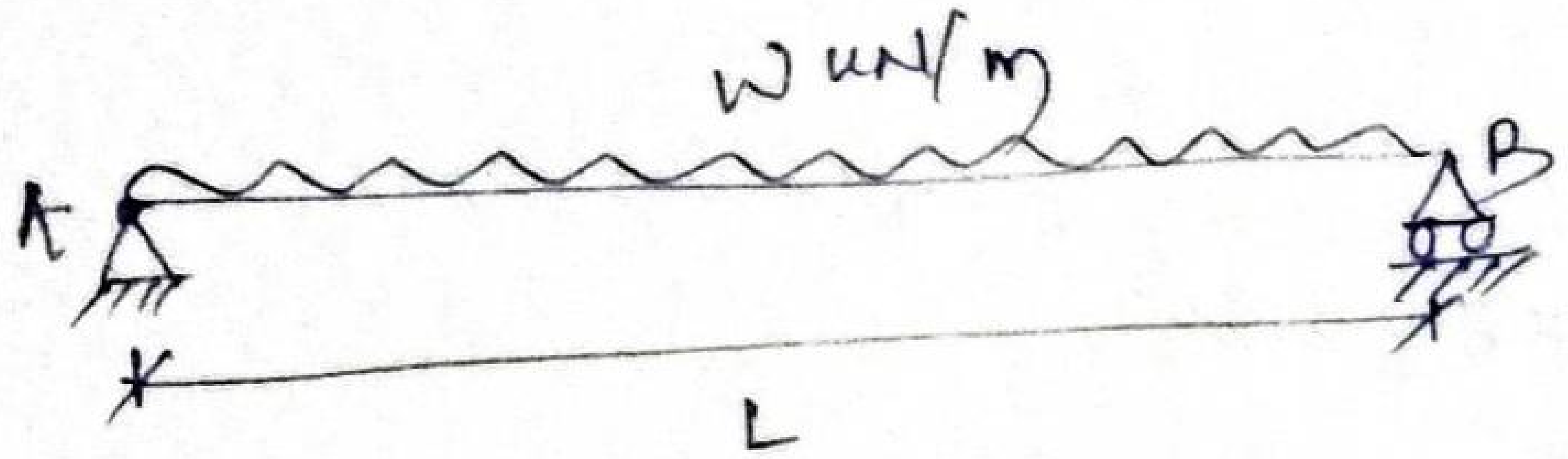
1) **Point Load :-** A point load is known as concentrated Load .



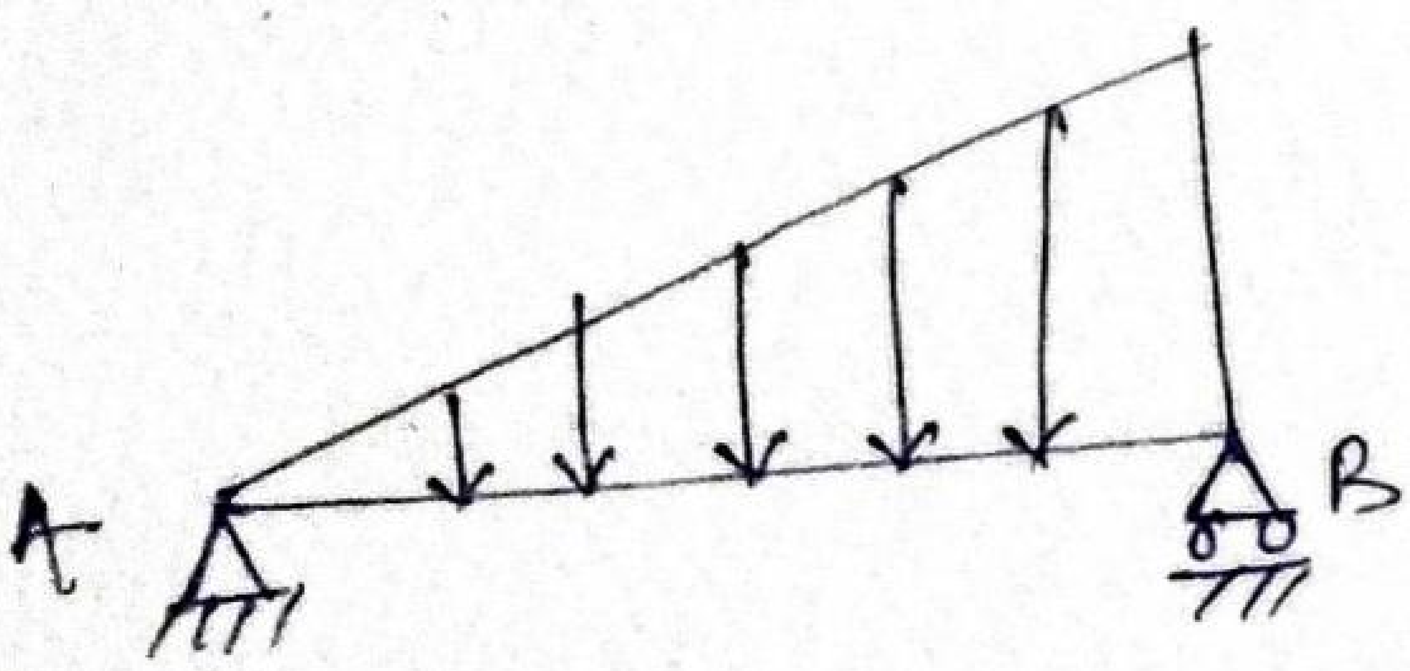
(Point Load)

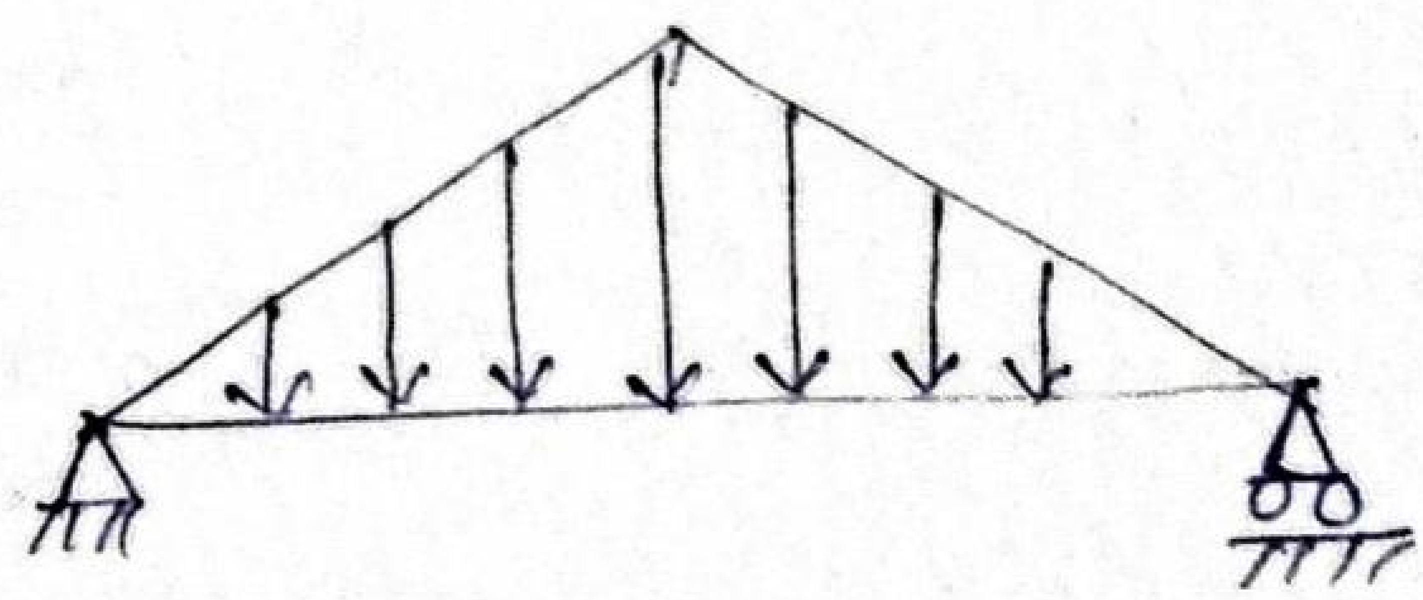
2) **uniformly distributed Load :-**

It is called as UDL . Uniformly distributed Load is special type of distributed Load in which intensity of loading is uniform say (W KN/m) .



(3) **uniformly varying load :-** Generally it is called UVL . UVL is a special type of distributed Load in which intensity of Load varies linearly .



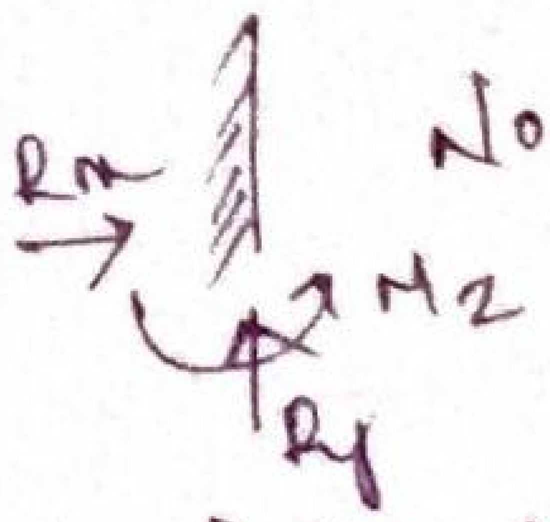


(4) Couple :- A concentrated moment at any point is known as couple

* Types of support

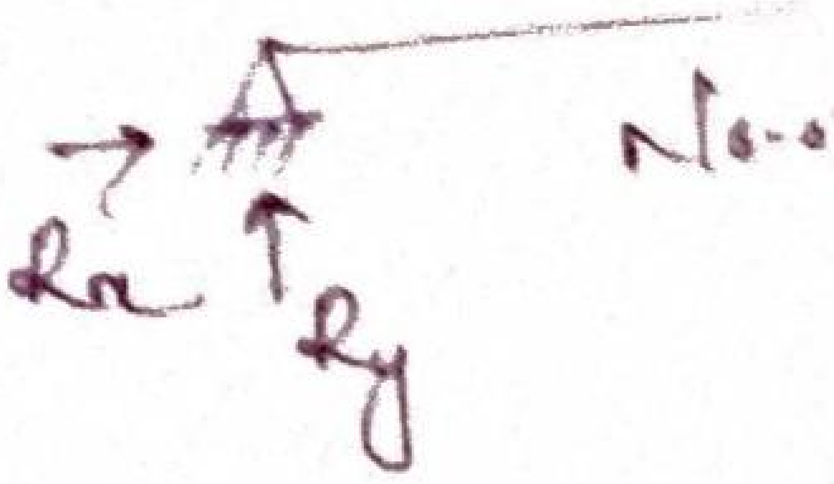
2D support

(a) fixed support :-



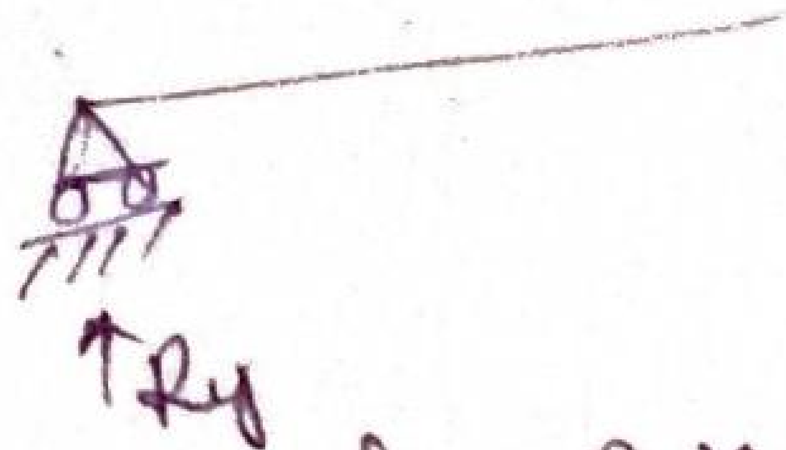
No. of reaction = 3

(b) Hinge support



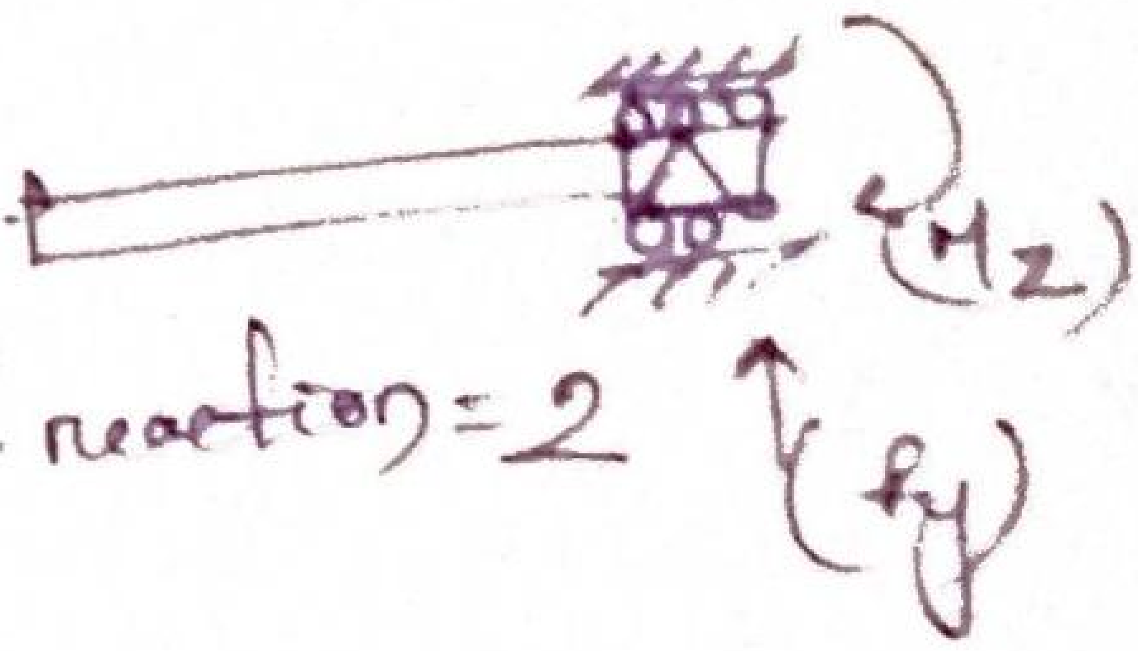
No. of reaction = 2

(c) Roller support



No. of reaction = 1.

(d) Double Roller support :-



At double roller support No. of reaction = 2

3D support :-

(a) fixed support :- At 3D fixed support there are 6 reactions

R_x R_y R_z

M_x M_y M_z.

(b) 3D hinged support :-

[No. of reaction = 3]

(c) 3D Roller support :- At 3D roller support there is only one reaction perpendicular to the plane of support.

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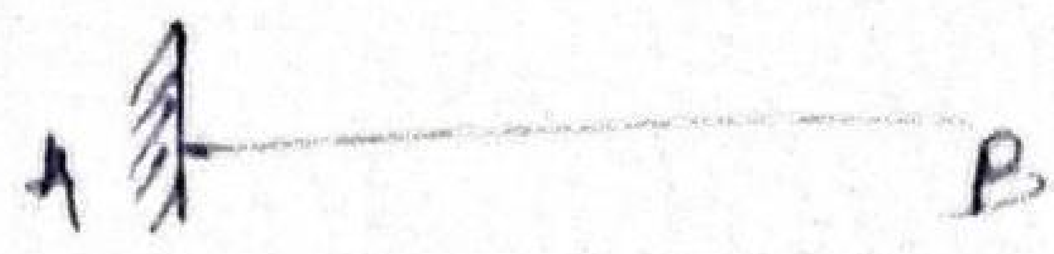
* Types of Beam

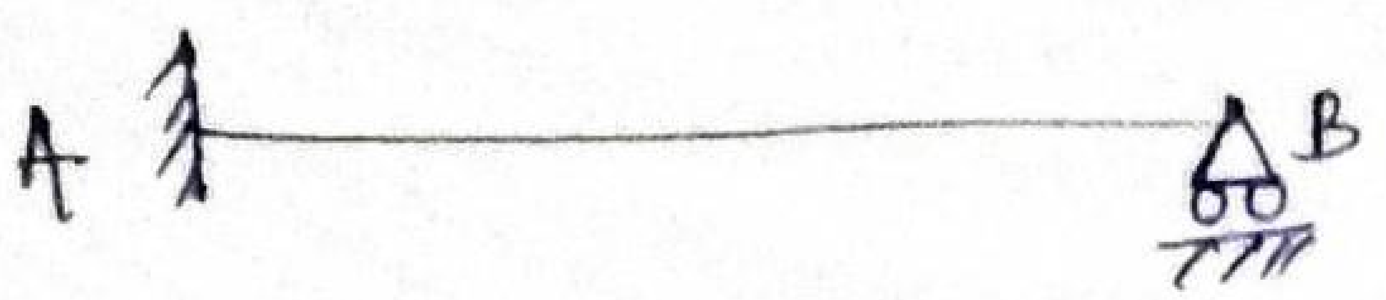
Beam is a structural member which is subjected to transverse loading to its axis.

(a) Simply supported beam

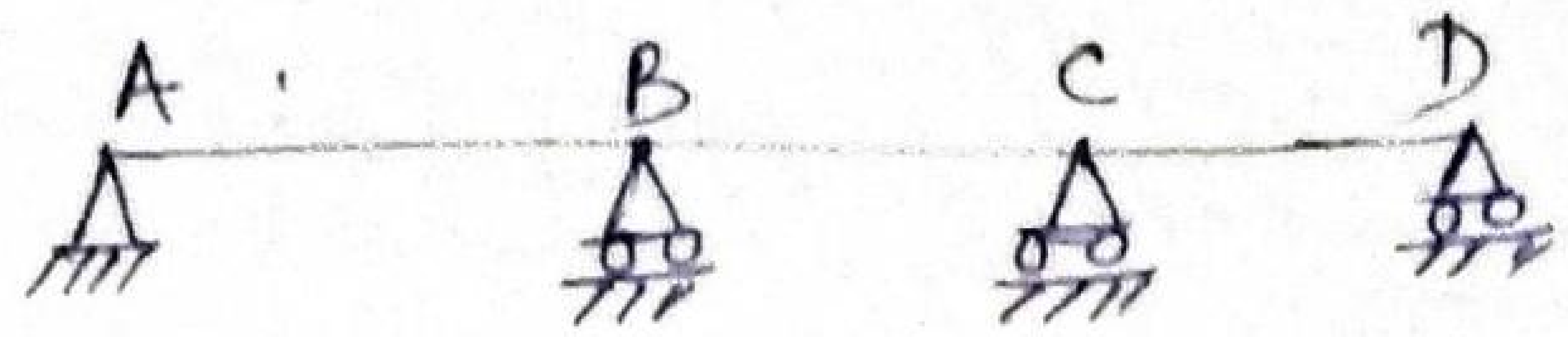
(b) cantilever Beam



(c) propped cantilever Beam



(d) continuous Beam



* Statically Determinate structure and indeterminate structure

A structure is said to be statically determinate if structure can be analysed by applying equation of static equilibrium ($\Sigma F_x = 0$, $\Sigma F_y = 0$ & $M_z = 0$).

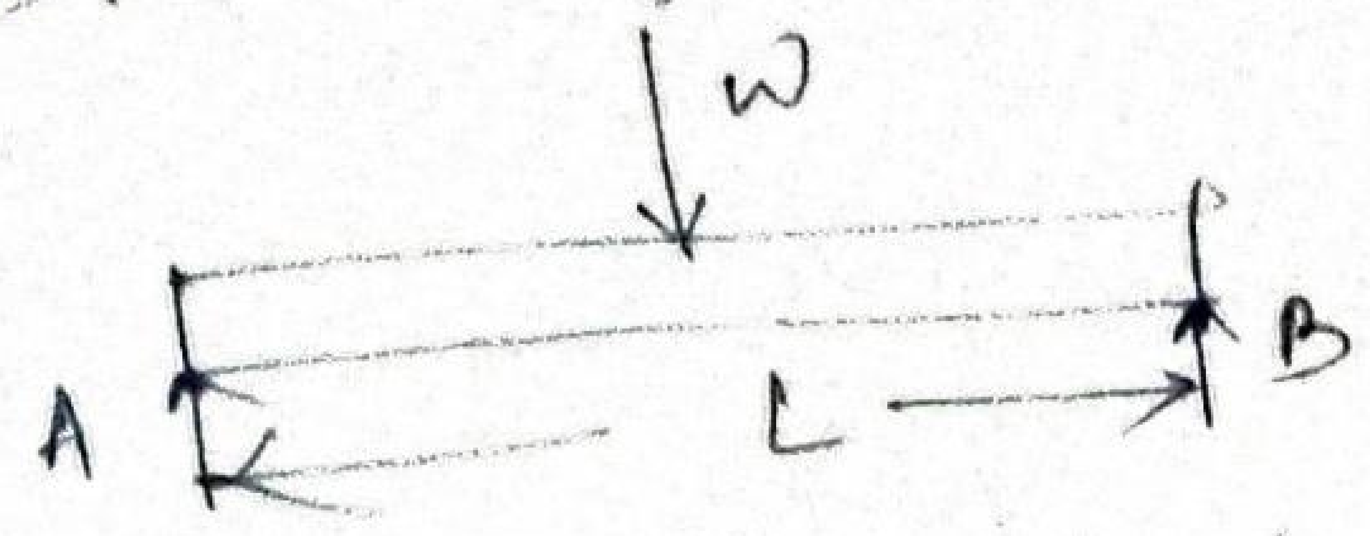
* If the structure can't analysed by using equation of equilibrium then that is known as statically indeterminate structure.

Formula (1) If $R = E$ (then structure externally determinate)

(2) If $R > E$ (Structure externally indeterminate).

What is Shearforce

We have a horizontal beam which is supported at its two ends. The length of beam is L.



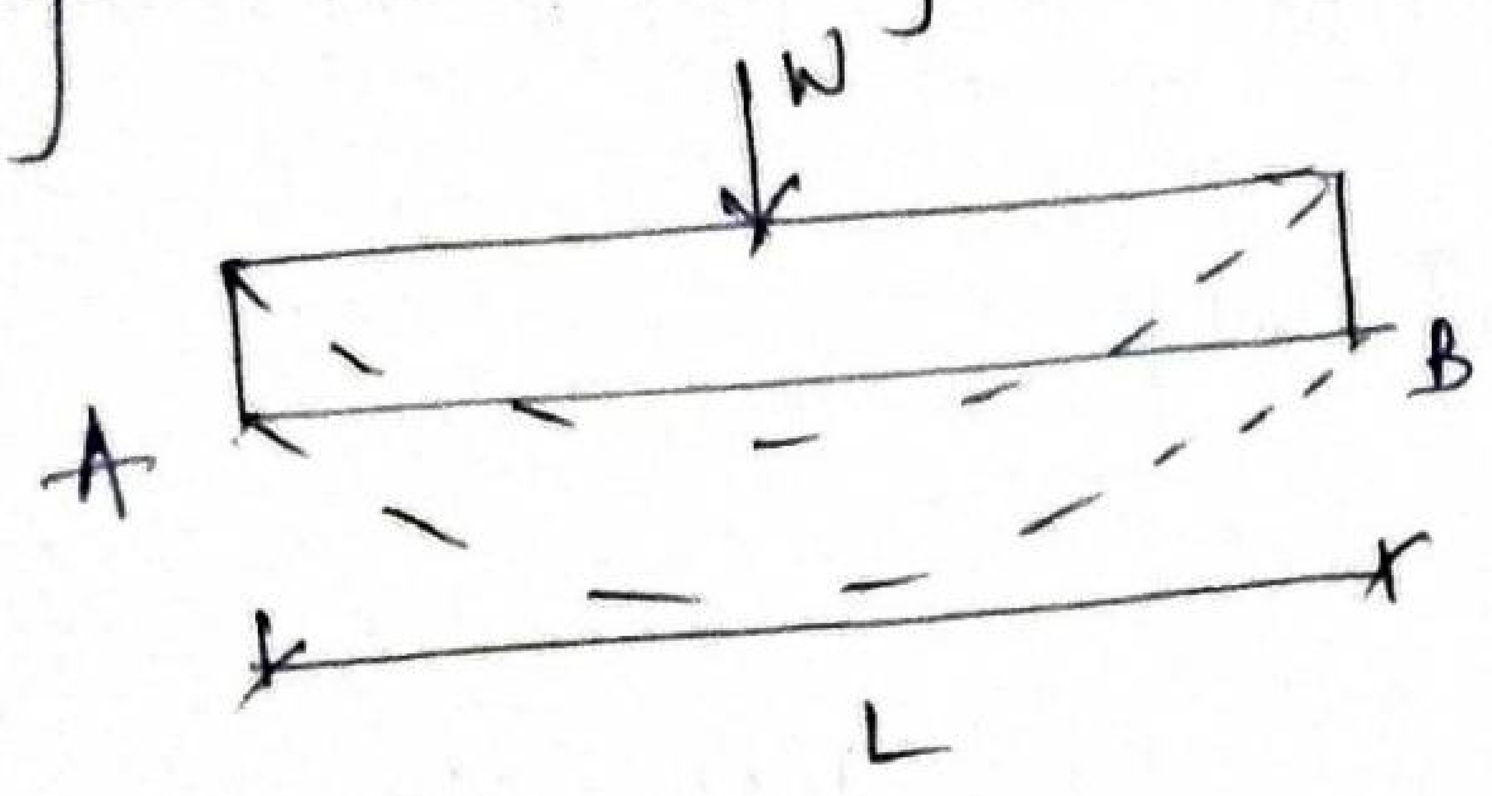
When we are loading this beam. Since the Load is acting downward and the beam is supported at two ends there is chances of this beam to break into two parts.

→ This breaking of beam is called as shearing action of beam.

→ The force because of which beams get sheared is called Shearforce.

Why Bending occurs in a beam:-

Suppose the beam is not shearing that means instead of shearing it is bending.



(Bending of beam)

Definition:-

Shearforce:- It is defined as the algebric sum of forces acting either on left handside or right handside of the section.

That means either add the forces on LHS or add forces on RHS of the section. That will give Shearforce value.

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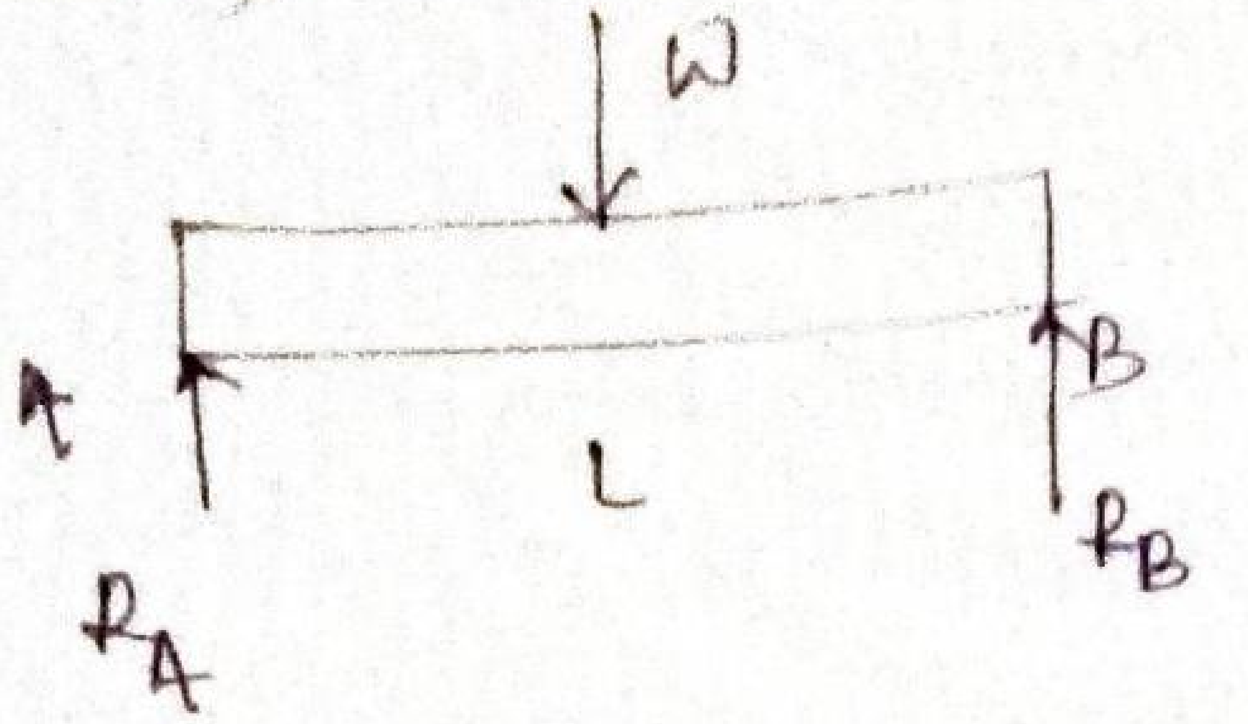
Bending Moment It is defined as the algebraic sum of Moments of forces acting on left side or right side of the section.



Moment from LHS
$= R_A \times L/2$

RHS $= R_B \times L/2$

In the above example we have only one force towards left or right.

→ The value of B.M is single. If we have more number of loading then we have to add the B.M of all the forces either to left or right.

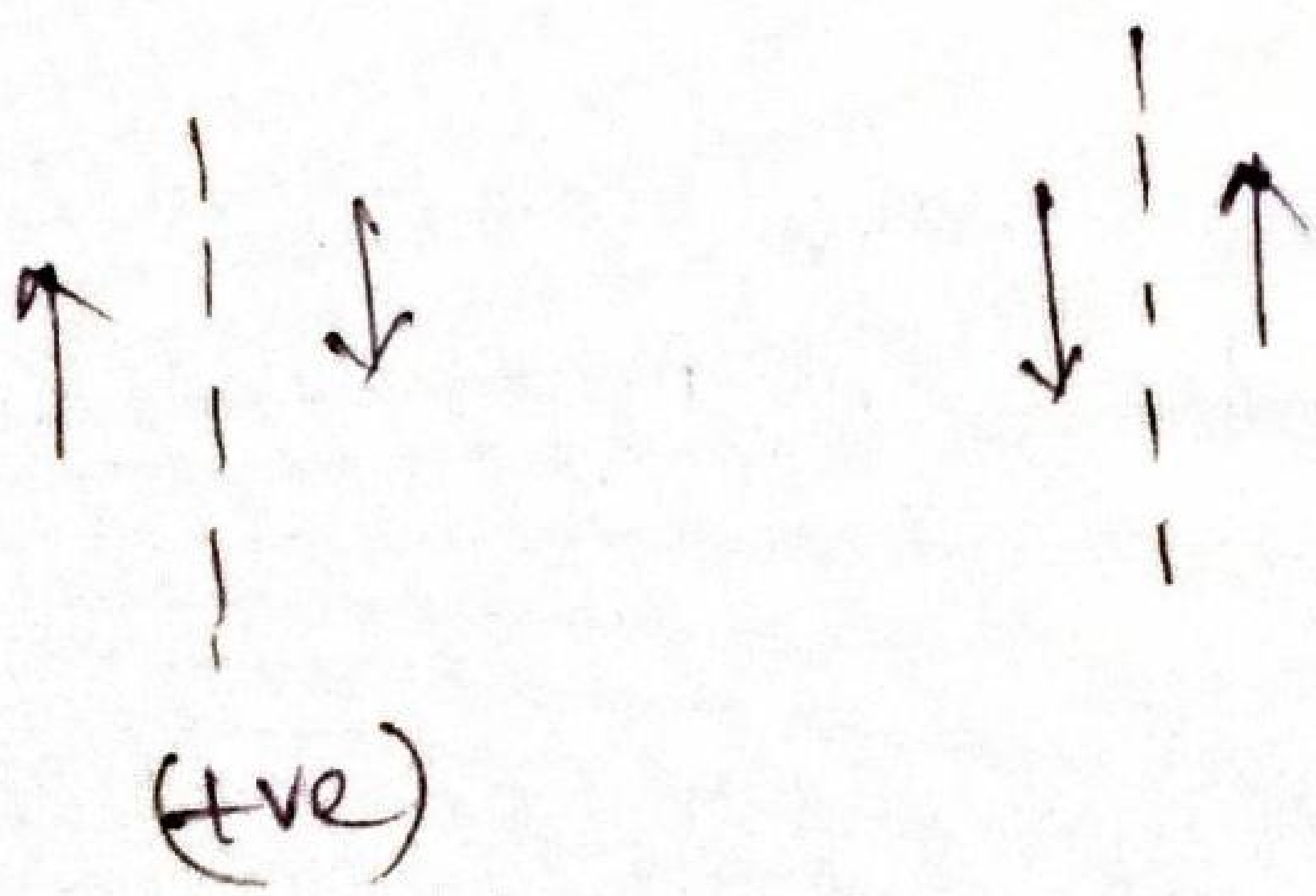
UNIT of S.F & B.M

S.F (Shear force) is denoted by [KN or N]

B.M (Bending moment) is denoted by [KNm or Nmm]

* **Sign convention of S.F and B.M**

Shear force



LU = +ve
RU = −ve
Rd = +ve
Ld = −ve.

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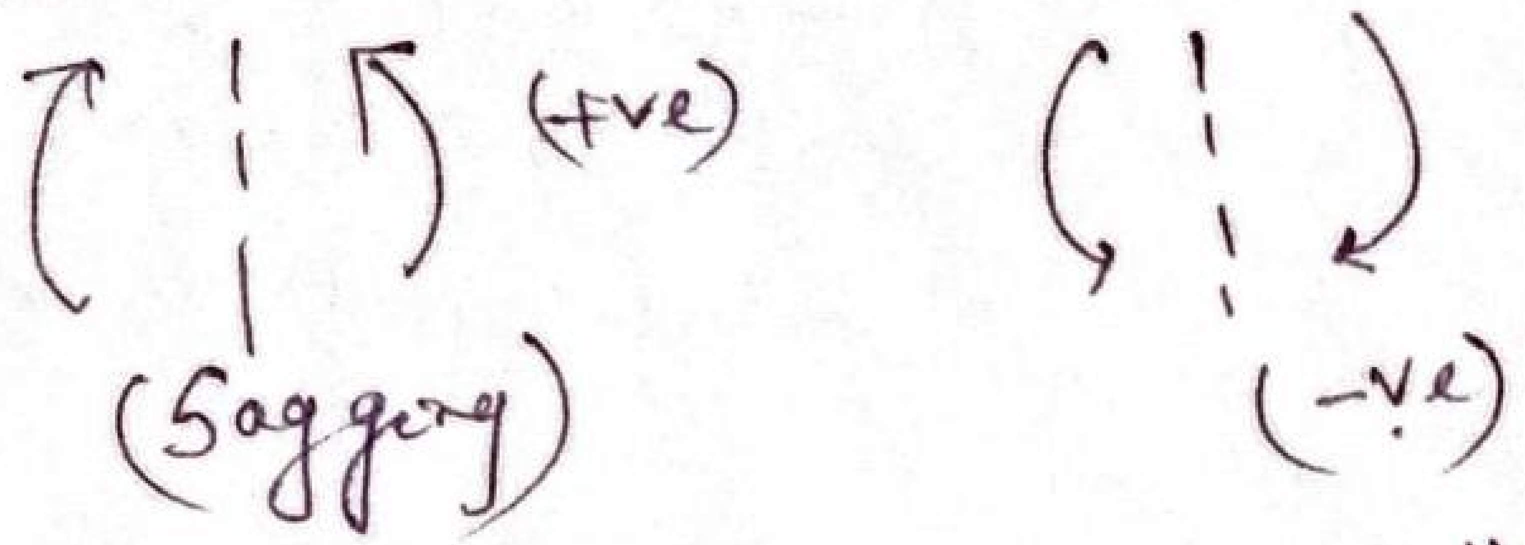
If a section is taken on a beam. Then to the left of the beam there is upward force and to the right of the section downward force.
Then the forces are taken as +ve.

Sign convention for bending moment

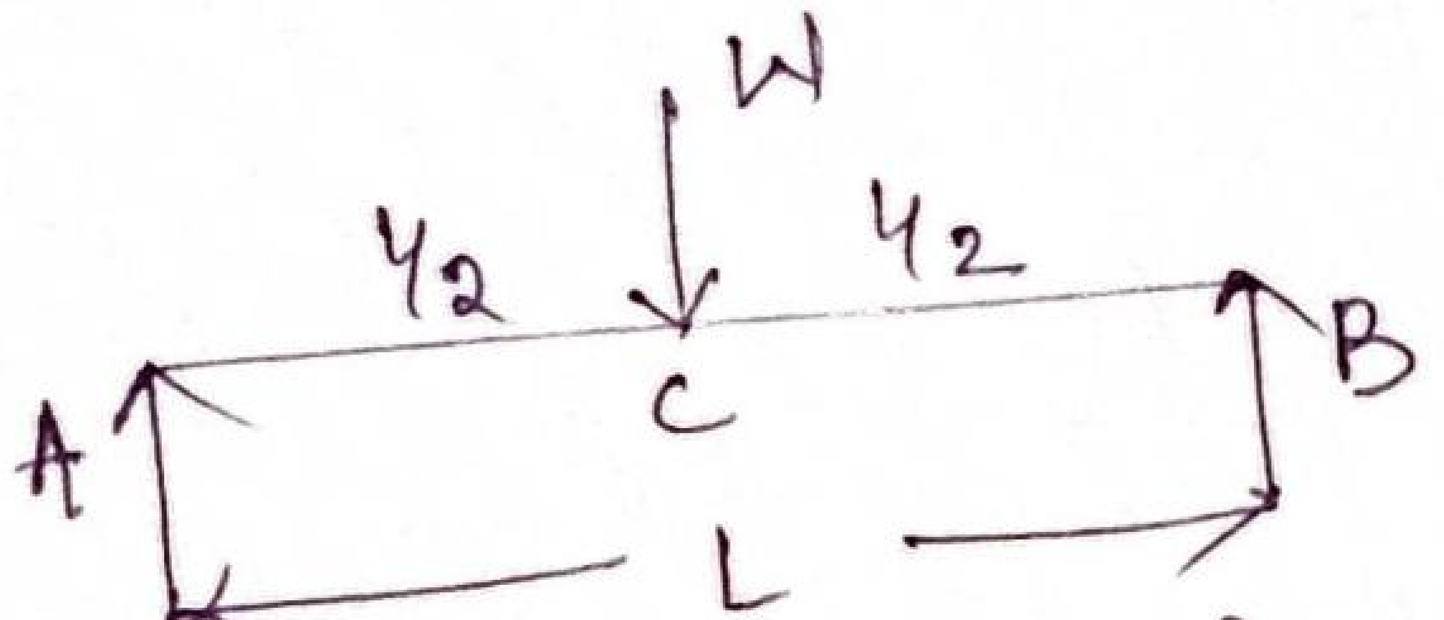


⇓ In sagging there is clockwise moment to left of section and anticlockwise to right of section.

⇓ In hogging there is anticlockwise moment to the left of the section and clockwise to the right of the section.

Case-1 :-

A Simply supported beam with a point load at its mid point



Consider a simply supported beam AB of span length 'L' and carrying a point load 'W' at its midpoint 'C'

→ Since the load is at the midpoint of the beam therefore the reaction at the support A

$$\boxed{R_A = R_B = W/2}$$

Calculation of support reaction

$\sum M_B = 0$

$\Rightarrow R_B \times l - w \times l/2 = 0$

$\Rightarrow R_B \times l = wl/2$

$\Rightarrow R_B = \frac{wl/2}{l} = w/2$

upward force = downward force

$\Rightarrow R_A + R_B = w$

$\Rightarrow R_A + w/2 = w$

$\Rightarrow R_A = w - \frac{w}{2} = \frac{w}{2}$.

$\boxed{R_A = w/2}$ & $\boxed{R_B = w/2}$

OR

- As this beam is symmetrically loaded due to vertical downward half load transferred to both support as $w/2$ & $w/2$

* Shearforce calculation :

$(S.F)$ at $A = +R_A = +w/2$

$(S.F)$ at $C = \frac{w}{2} - w = -w/2$

$(S.F)$ at $B = -w/2 + w/2 = 0$.

* Bending moment calculation : (+) (−)

$(B.M)_A = 0$

$(B.M)_C = R_A \times l/2 = w/2 \times l/2 = wl/4$.

$(B.M)_B = 0.$

Note :- At endpoints A and B = 0 because there is no force acting at both the end points.

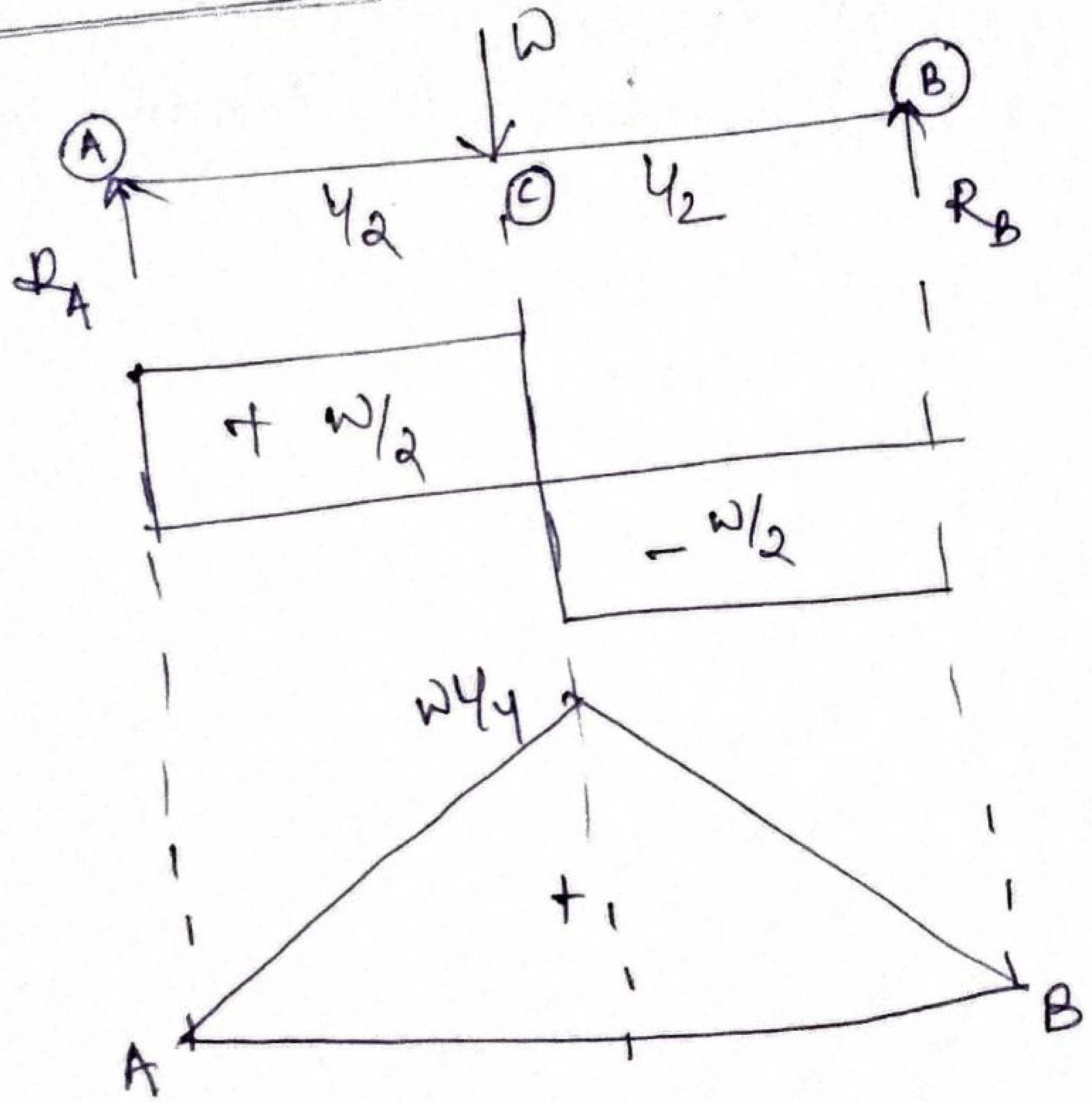
→ consider all forces acting at LHS of C. R_A is acting clockwise Moment before 'C'

from A to C = distance ($l/2$)

$$\text{Moment} = R_A \times l/2$$

$$= W/2 \times l/2 = Wl/4.$$

SFD and BMD



In case of point load inclined straight line to be presented in case of BMD.

* Important points to be noted while drawing SFD and BMD

(1) length of SFD and BMD must be equal to the span of the beam.

(2) S.F.D is drawn below the loaded beam and BMD is drawn below SFD.

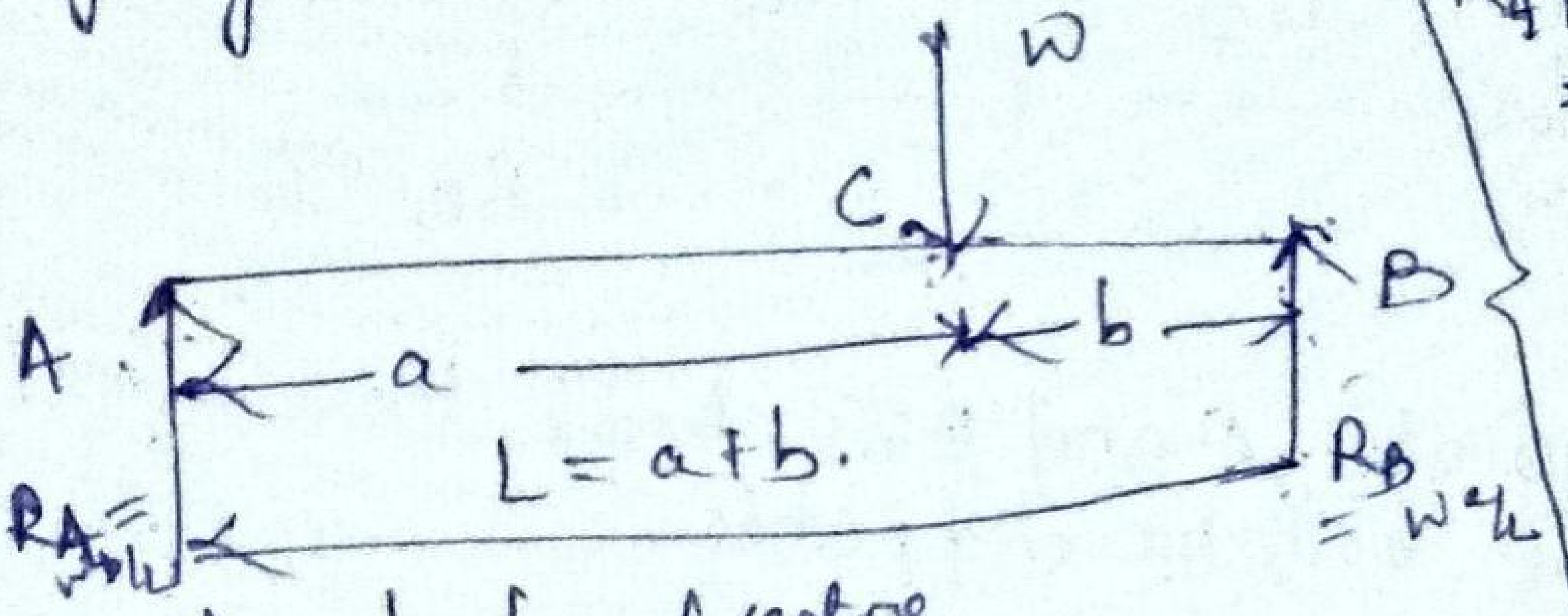
(3) For simply supported beam B.M is zero at the support.

(4) for cantilever beams B.M will be zero at the free end.

(5) Calculate S.F and B.M at all critical points.

(6) If no load is present between two points then S.F will be constant.

Case-2 :- A simply supported beam carrying an eccentric point load.



$R_A + R_B = W$

$\Sigma M_A = 0$

$R_B \times L - W \times a = 0$

$R_B \times L = W \times a$

$R_B = \frac{Wa}{L}$

$R_A + \frac{Wa}{L} = W$

$R_A = W - \frac{Wa}{L}$

$= \frac{Wb}{L}$

The Load is not acting at centre

$R_A = Wb/L = $ S.F at A, S.F at B $= Wa/L = R_B$

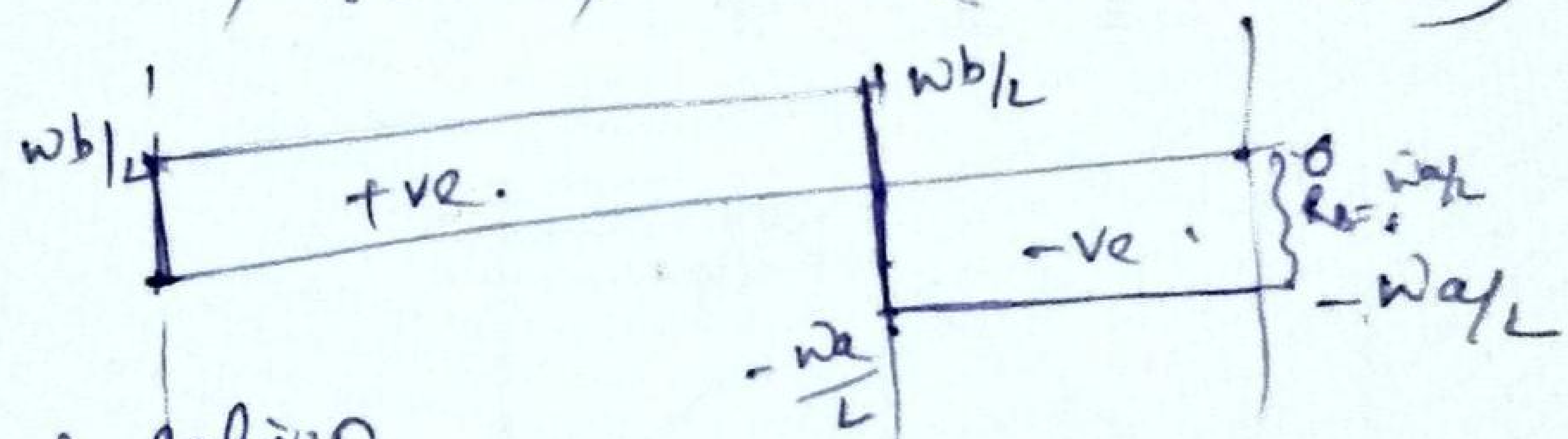
S.F at C $= \frac{Wb}{L} - W = W\left[b/L - 1\right]$

$= W\left[\frac{b-L}{L}\right]$

$= -W\left[\frac{L-b}{L}\right] = -\frac{Wa}{L}$ $\because (L-b=a)$

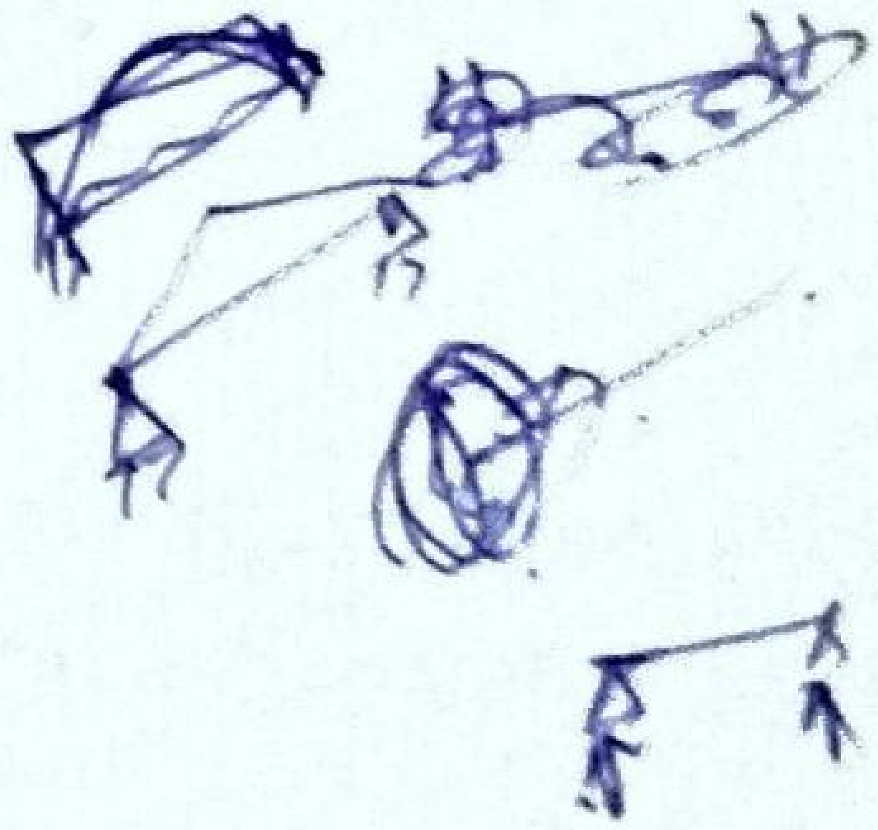
S.F at B (Left) $= -Wa/L$ (as no force acting B & C)

S.F at B $= -\cancel{W}a/L + Wa/\cancel{L} = 0$. ($Wa/L$ = +ve upward force)



B.M.D calculation

(clockwise = +ve) (anticlock -ve)



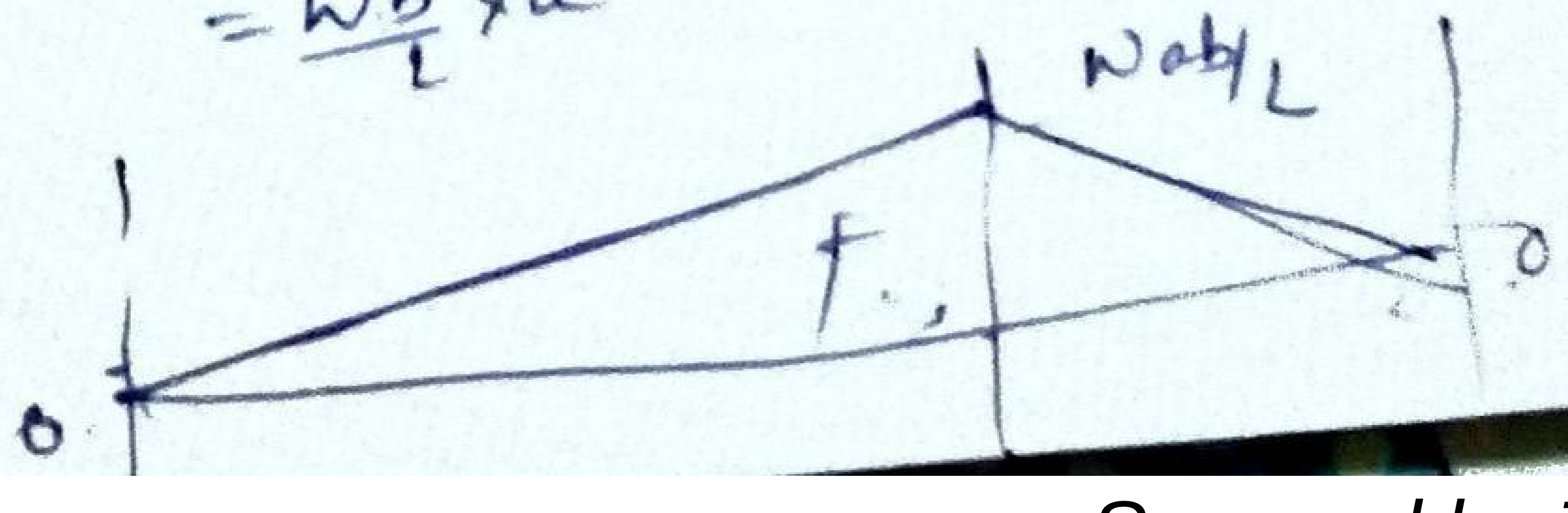
$(B.M)_A = 0$

$(B.M)_B = 0$

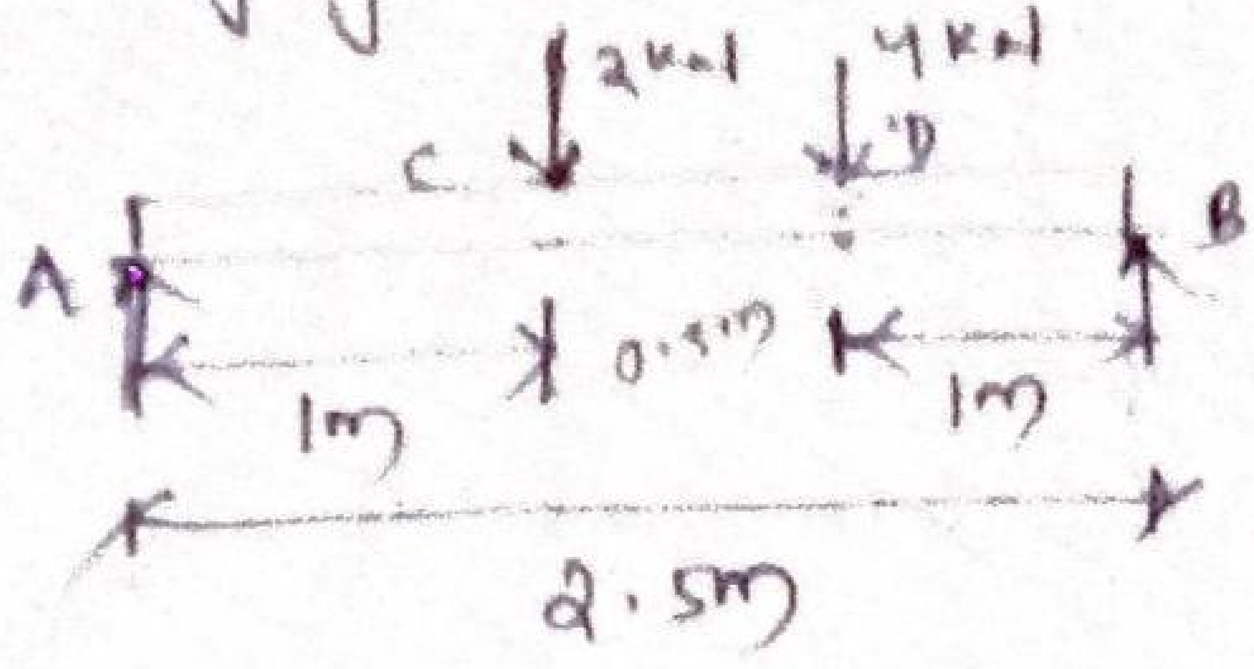
$(B.M)_C = R_A \times \perp^r$ distance

$= R_A \times a$

$= \frac{Wb}{L} \times a = Wab/L$.



Q. A Simply supported beam AB, of span 2.5m is carrying two point loads as shown in fig.



draw the S.F and B.M diagrams.

Ans.

given span (l) = 2.5m.

point load at C = W_1 = 2kN

Pt. load at B = W_2 = 4kN.

first of all let us find out the reaction R_A and R_B. Taking moment about A and equating the same.

$\boxed{R_A + R_B = 2+4}$ as upward force = downward force

$$\Sigma M_A = 0$$

$$R_B \times 2.5 = (2\times1) + (4\times1.5)$$

$$R_B \times 2.5 = 8$$

$$\Rightarrow \boxed{R_B = 8/2.5 = 3.2 \text{ kN}} \quad \boxed{R_B = 3.2\text{kN}}$$

$$R_A + R_B = 2+4$$

$$\Rightarrow R_A + (3.2) = 6$$

$$\boxed{R_A = 6 - 3.2 = 2.8\text{kN}.} \quad \boxed{R_A = 2.8\text{kN}}$$

Shear force calculation

S.F at A $F_A = +R_A = 2.8$kN.

S.F at C = $F_C = 2.8 - 2 = 0.8$ kN.

S.F at D = $F_D = 0.8 - 4 = -3.2$ kN.

$F_B = -3.2$kN. (as no load betn D and B)

Bending moment calculation :- as simply supported beam. B.M at A (M_A) = 0.

$$M_C = R_A \times 1 = 2.8 \times 1 = 2.8\ \text{kNm}$$

$$M_D = R_B \times 1 = 3.2 \times 1 = 3.2\ \text{kNm}$$

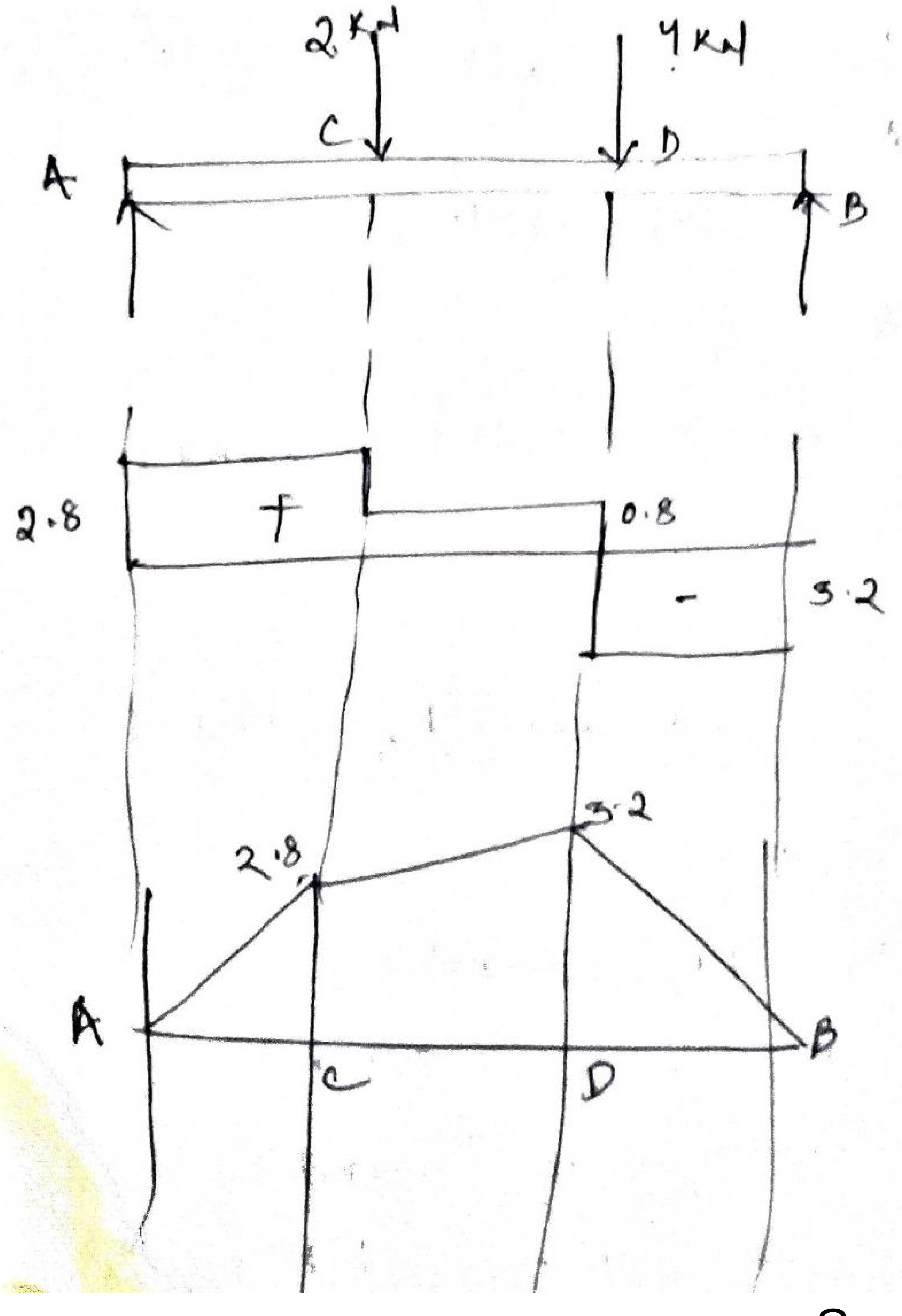
$$M_B = 0.$$

The value of M_D may be found from one end (R_A)

$$\Rightarrow M_D = R_A \times 1.5 - 2 \times 0.5$$

$$= (2.8 \times 1.5) - (2 \times 0.5) = 4.2 - 1$$

$$= 3.2\ \text{kNm}.$$



* Important points to be noted while drawing SFD and BMD

(1) Length of SFD and BMD Must be equal to the span of the beam.

(2) S.F.D is drawn below the Loaded beam and BMD is drawn below SFD.

(3) For simply supported beam B.M is zero at the support.

(4) for cantilever beams B.M will be zero at the free end.

(5) Calculate S.F and B.M at all critical points.

(6) If no load is present between two points then S.F will be constant.

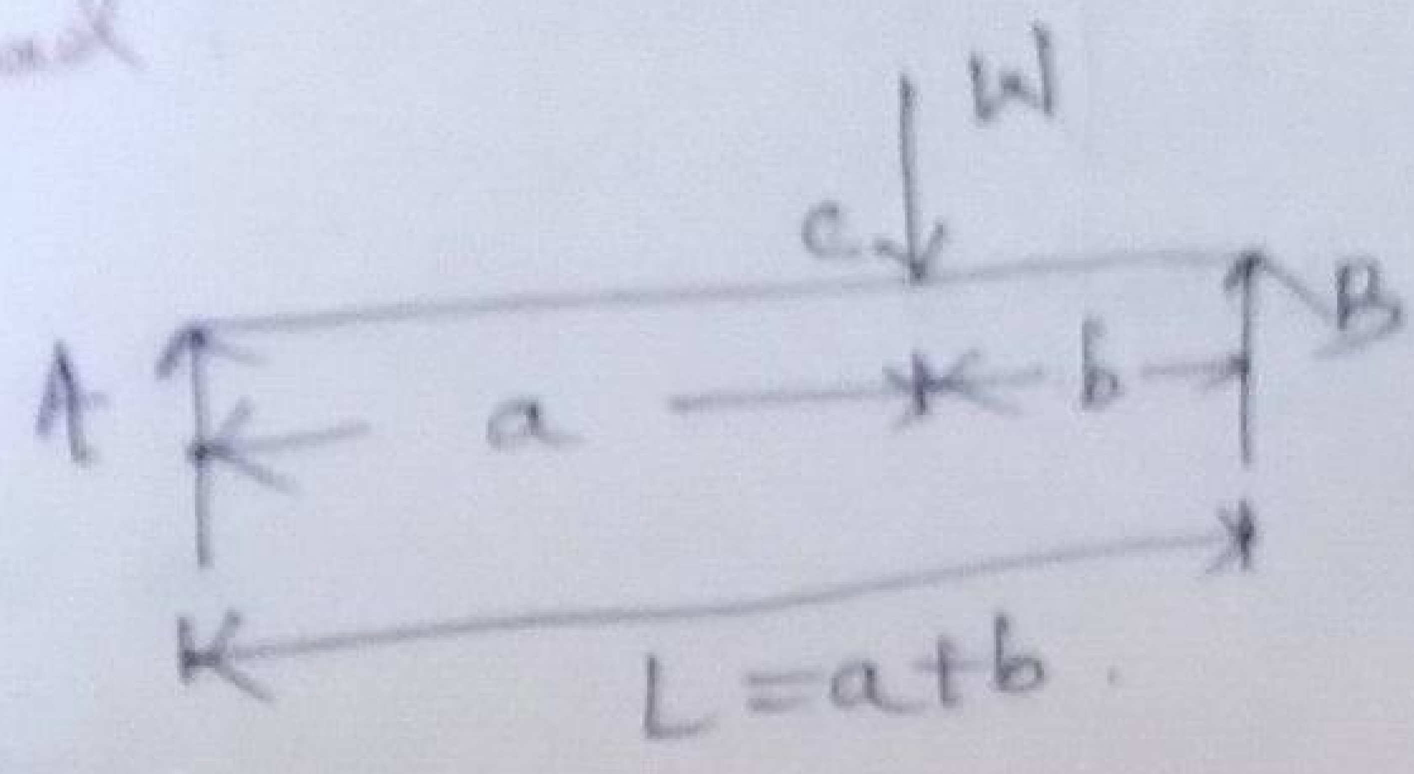
Q: Why we calculate shear force and Bending Moment?

A:- Structures fails when ever it is not in the state of equilibrium.

→ If forces and couples are acting on a structure is stable and no failure occurs.

But shear force and Bending Moment are nothing but unbalanced forces and moments respectively. So in order to make our structure durable and to satisfy its requirements, we need to take care of these unbalanced forces and couples and we have to know its measure to analyze the structure.

Case-2: A simply supported beam carrying an eccentric point load



Step-1

Support reaction calculation

$\Sigma M_A = 0$

$\Rightarrow R_B \times L - W \times a = 0$

$\Rightarrow \boxed{R_B = Wa/L}$

upward force = downward force

$\Rightarrow R_A + R_B = W$

$\Rightarrow R_A + Wa/L = W$

$\Rightarrow R_A = W - Wa/L = Wb/L$

$\boxed{R_A = Wb/L}$

Step-2:- Shear force calculation

$R_A = $ S.F at A $= Wb/L$.

S.F at B $= R_B = Wa/L$.

S.F at C $= R_A - W = \frac{Wb}{L} - W = W\left[b/L - 1\right]$

$= -W\left[\frac{L-b}{L}\right] = -Wa/L \qquad \left[\because L-b=a\right]$

S.F at B (left of B) = $-Wa/L$ (as no force bet.n B & C).

S.F at B = $\boxed{-Wa/L + Wa/L = 0}$.

Step-3: Bending Moment calculation.

B.M at A = 0

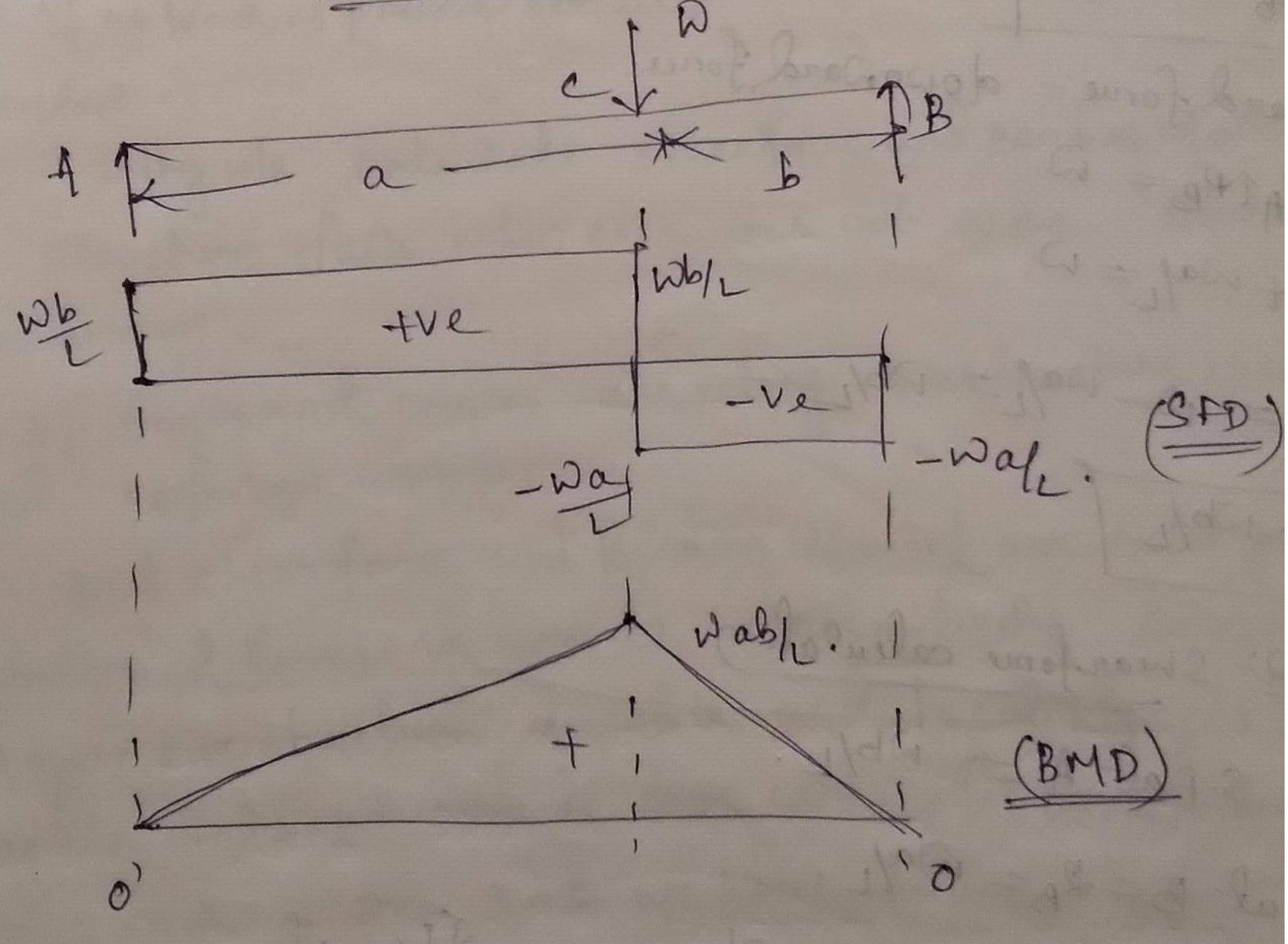
B.M at B = 0.

B.M at C = $R_A \times$ $\perp^r$ distance

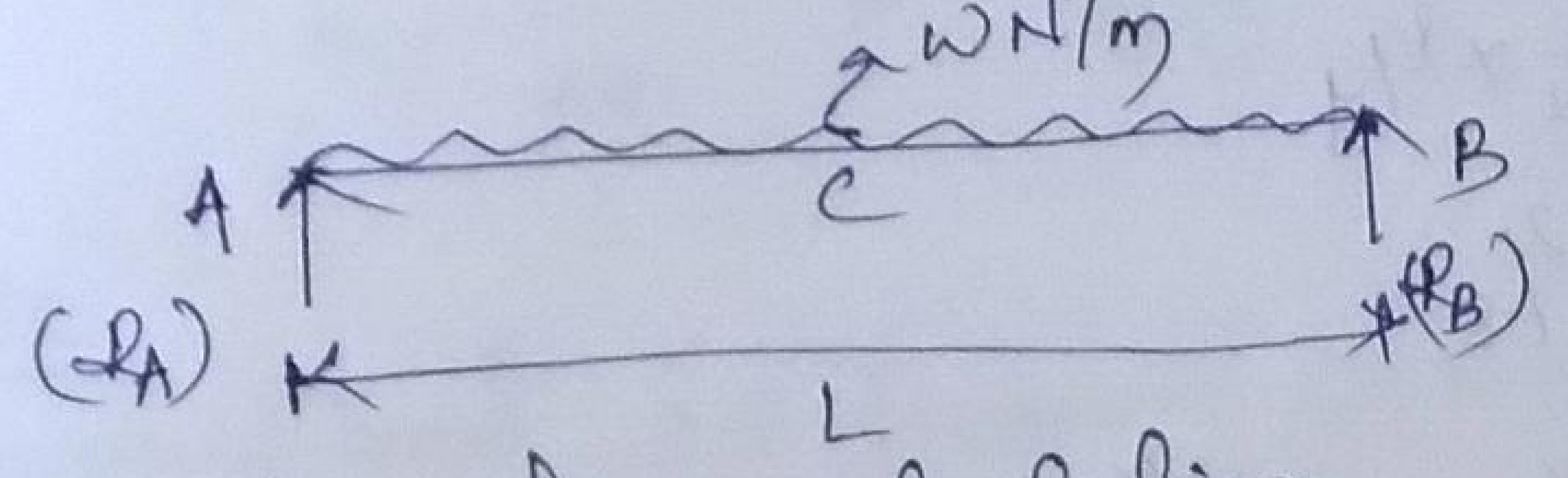
$= R_A \times a$

$= Wb/L \times a = Wab/L$.

Step-4: S.F.D and B.M.D



Case-5 :- A simply supported beam carrying a udl of w N/m over the entire span.



Support reaction calculation

Taking moment about A $\Sigma M_A = 0$.

$\Rightarrow R_B \times L - w \times L \times L/2 = 0$

$\Rightarrow R_B \times L = wL^2/2$

$\Rightarrow R_B = \frac{wL^2/2}{L} = wL/2$

$\Rightarrow \boxed{R_B = wL/2}$

upward force = downward force

$\Rightarrow R_A + R_B = wL$

$\Rightarrow R_A + wL/2 = wL$

$\Rightarrow R_A = wL - wL/2 = wL/2$

$\boxed{R_A = R_B = wL/2}$

Shear force calculation

S.f at A = $wL/2$

S.f at C = $wL/2 - \frac{wL}{2} = 0$

S.f at B (left of B) = $\frac{wL}{2} - wL = -wL/2$.

S.f at B = $-wL/2 + wL/2 = 0$.

Banding Moment calculation.

B.M at A = 0.

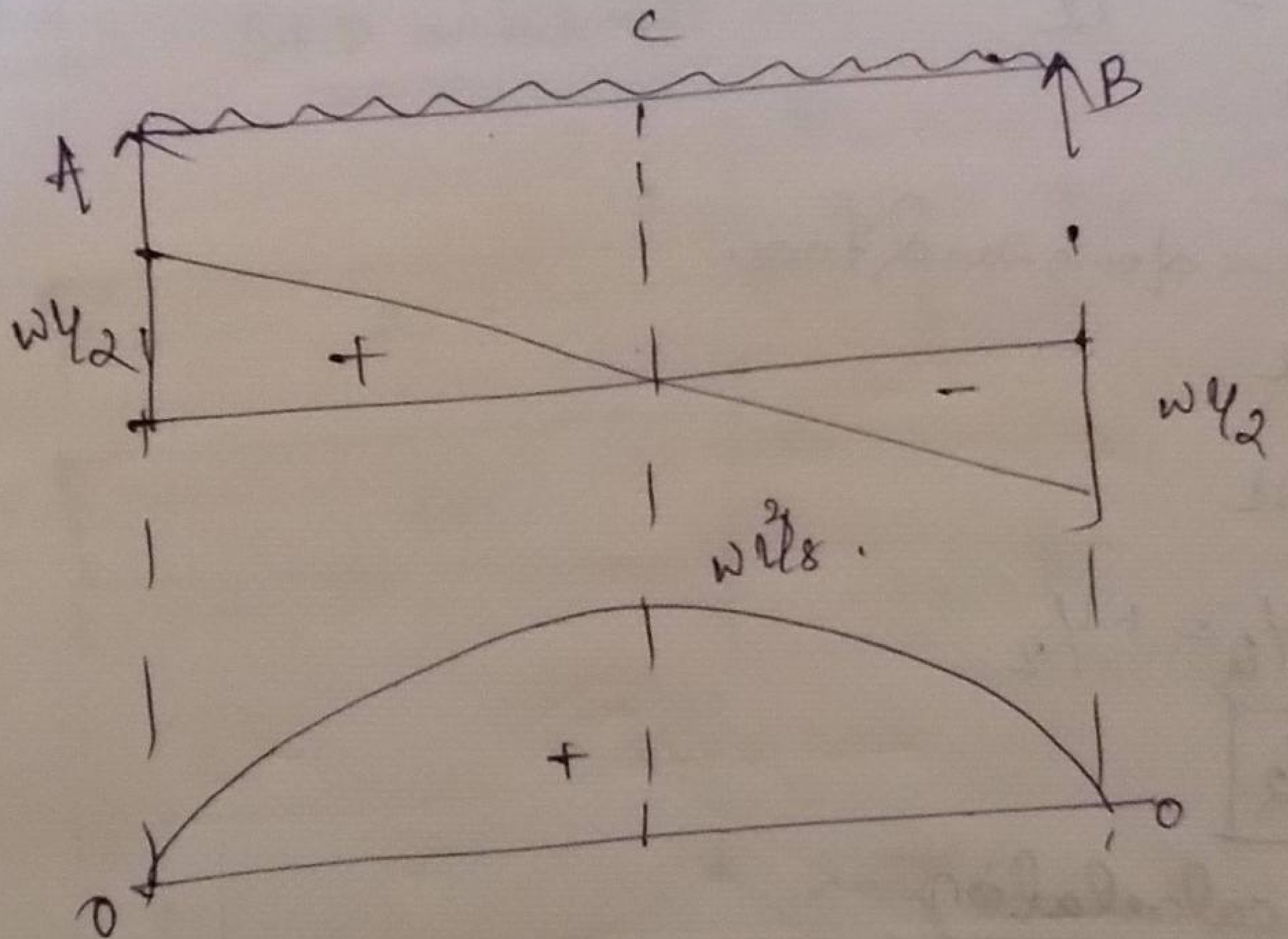
B.M at C = $R_A \times L/2 - \frac{wL}{2} \times L/4$

$= \frac{wL}{2} \times L/2 - wL^2/8$

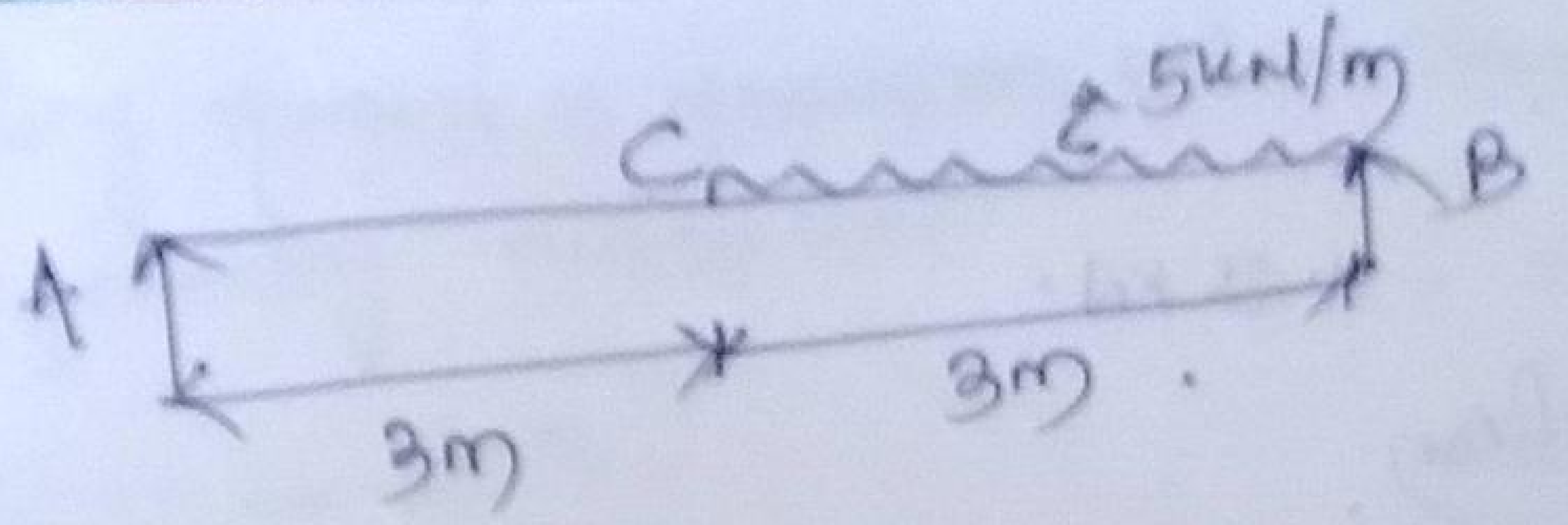
$= \frac{wL^2}{4} - wL^2/8 = wL^2/8$.

B.M at B = $\frac{wL}{2} \times L - wL \times L/2 = 0$.
(from left)

As well as for a simply supported beam at end support B.M = 0



Q. A simply supported beam 6m long is carrying a udl of 5KN/m over a length of 3m from the right end. Draw S.F and B.M diagram for the beam & also calculate the maximum B.M on the section.



Note
for point load s.f diagram will straight line and for B.M = inclined line.
for uniformly distributed Load shear force = inclined diagram
& Bending moment diagram = parabolic.

given data

Span $L = 6m$
udl $(w) = 5kN/m$.

$\Sigma M_A = 0$.

$\Rightarrow R_B \times 6 - 5 \times 3 \times (3/2 + 3) = 0 \rightarrow (w \times L/2 \times L/4 + span)$

$\Rightarrow R_B = \frac{15 \times 4.5}{6} = 11.25 kN$.

$R_A + R_B = 5 \times 3$

$\Rightarrow R_A + 11.25 = 15$

$\Rightarrow \boxed{\uparrow R_A = 3.75 kN}$

Shear force calculation

S.F at A $= F_A = +R_A = +3.75 kN$.

S.F at C $= F_C = +3.75 kN$ (as there is no Load betⁿ A & C)

S.F at B $= R_A - 5\times3$.

$= 3.75 - 15 = -11.25$ KN.

Bending moment calculation.

B.M at A = 0.

B.M at C $= R_A \times l/2$

$= 3.75 \times 3 = 11.25$ KN.

B.M at B $= R_A \times l/2 - 5\times3\times3/2$

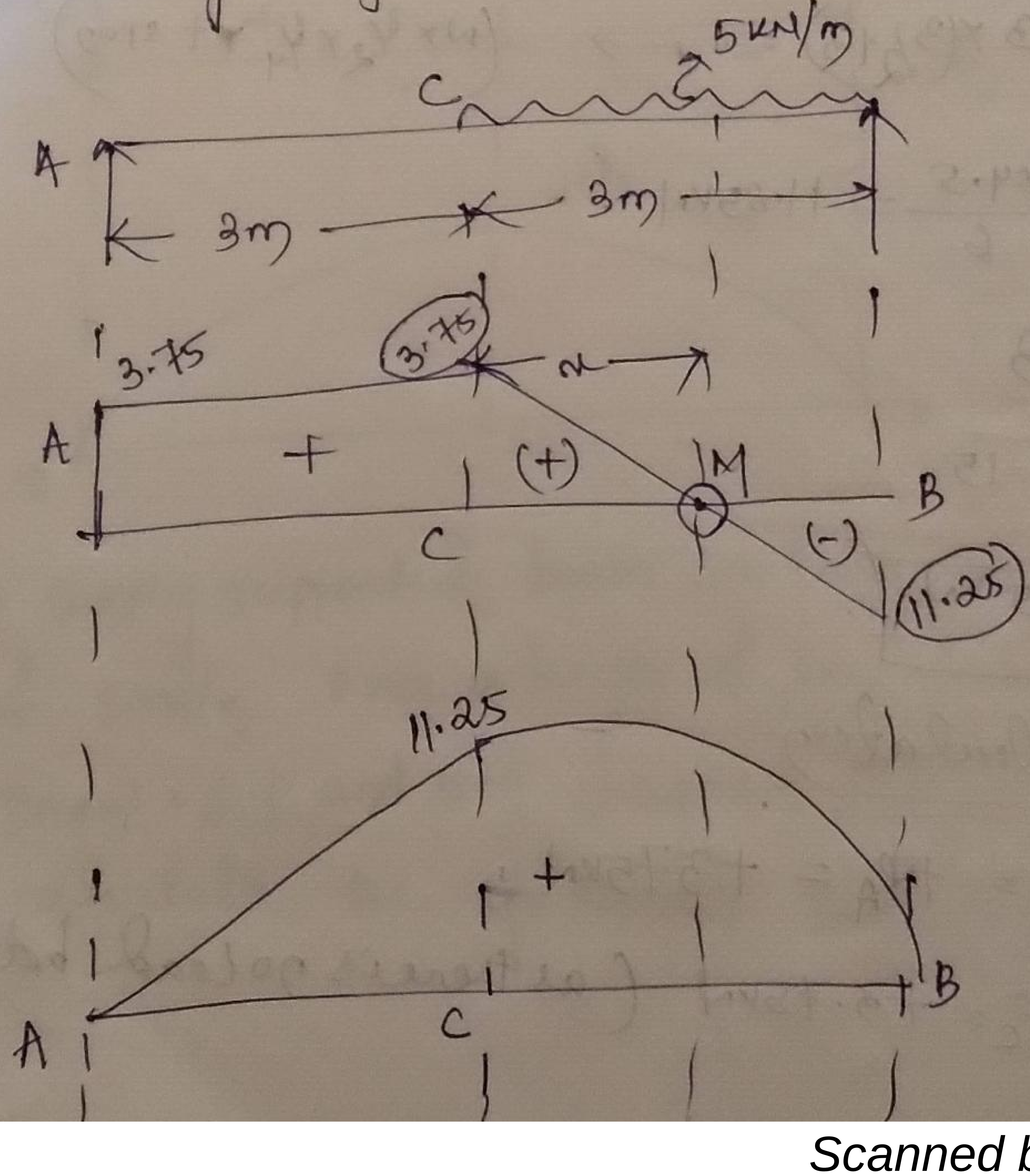
$= 3.75 \times 6 - 5\times3\times1.5 =$

$= 22.25 - 22.5 = 0$.

$R_A \times l - 5\times3\times3/2$

$\Rightarrow 22.5 - 22.5 = 0$

Note We know that Maximum B.M will occour at 'M' where S.F changes sign.



Bending moment is maximum where shear force changes sign.
from geometry we find that

$$\frac{x}{3.75} = \frac{3-x}{11.25}$$

$\Rightarrow 11.25x = (3-x)\,3.75$

$\Rightarrow 11.25x + 3.75x = 11.25$

$\Rightarrow 15x = 11.25$

$\Rightarrow x = \frac{11.25}{15} = 0.75\,m.$

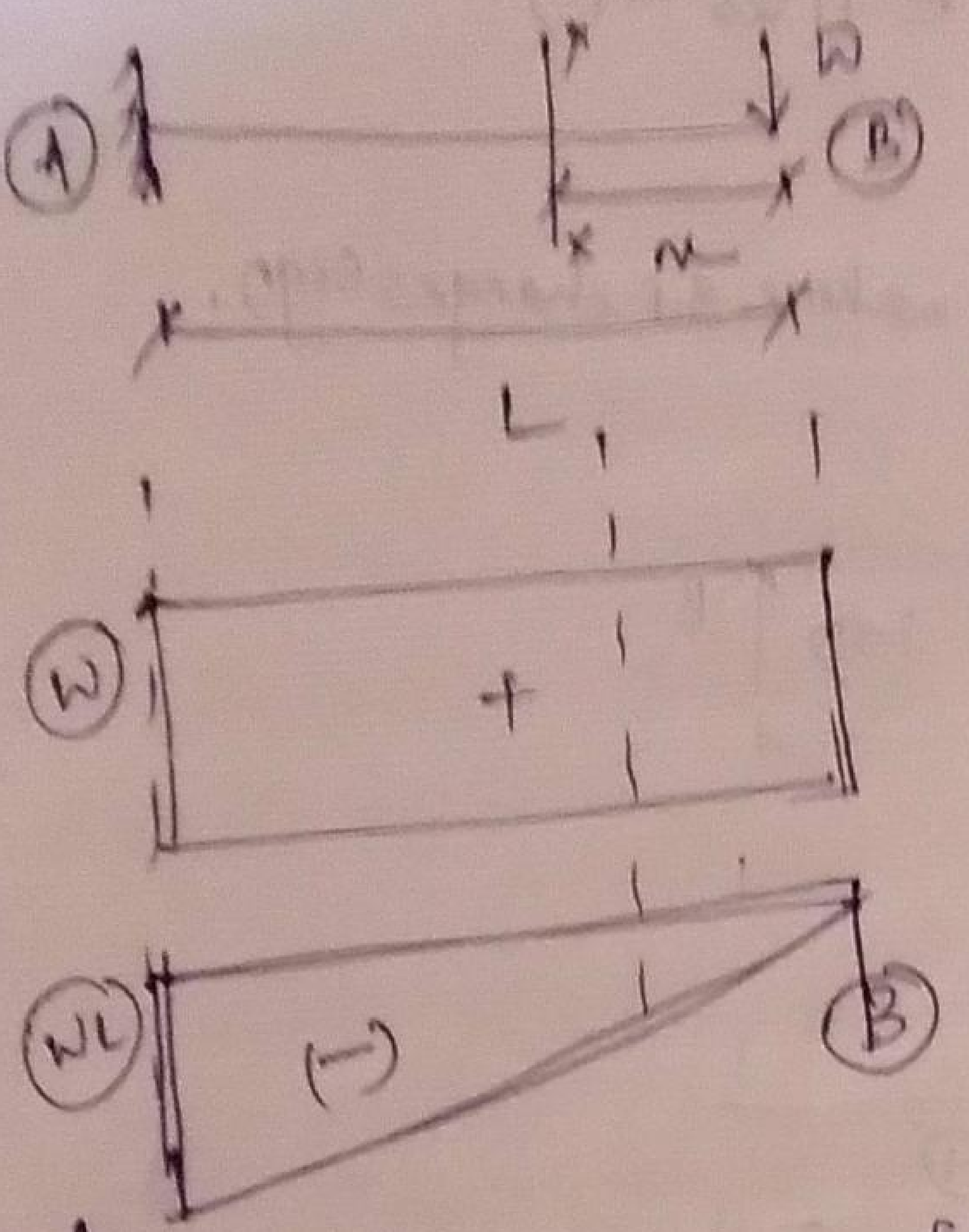
Maximum B.M at point M

$= R_A \times (3+0.75) - 5 \times 0.75 \times \frac{0.75}{2}$

$= 12.66\,KNm.$

* Cantilever with a point load at its free end

Consider a cantilever beam AB of length 'L' and carrying a point load 'w' at its free end.



S.F $\begin{cases} LU = +ve \\ Ld = -ve \\ RU = -ve \\ Rd = +ve \end{cases}$

B.M $\begin{cases} LC = +ve \\ La = -ve \\ RC = -ve \\ Ra = +ve \end{cases}$

We know that the shear force at any section 'x' at a distance 'x' from the free end is equal to total unbalanced vertical force.

$$\boxed{F_x = +W}$$

B.M at $x = -Wx$ (hogging)

When $x = L$

B.M at $A = -W \times L = -WL$.

B.M at $B = 0$.

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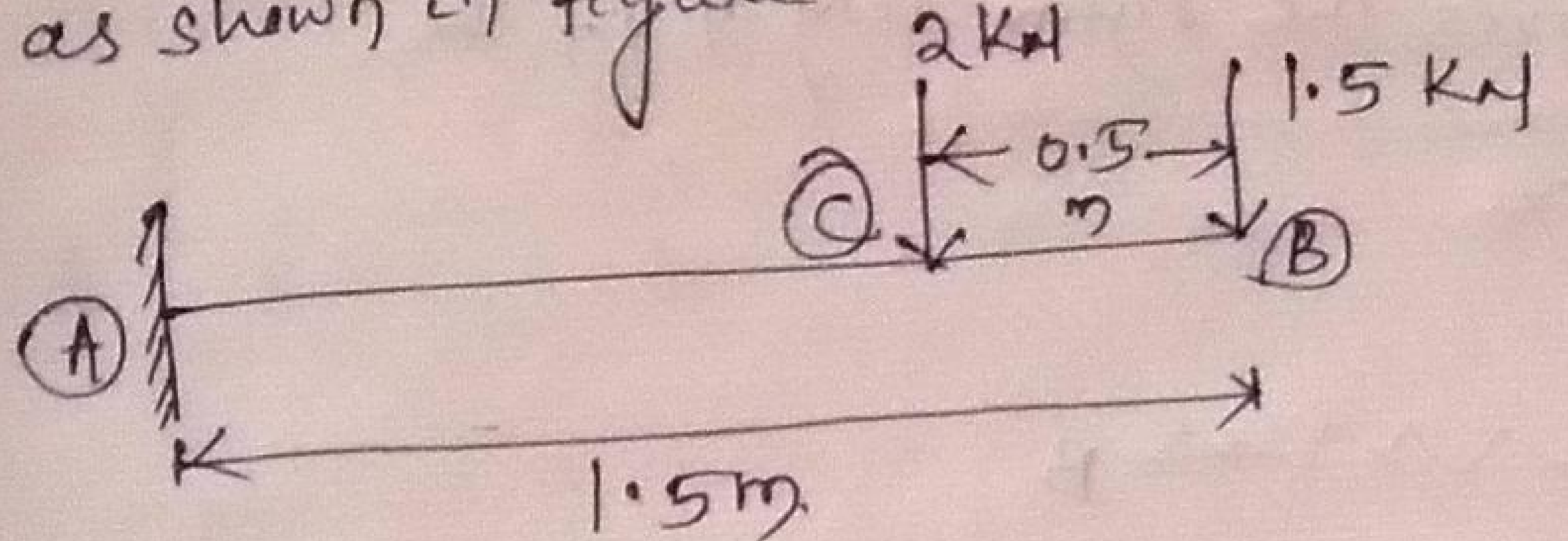
Q. Draw shearforce and Bending moment diagram for a cantilever beam of span 1.5m carrying point loads as shown in figure.



Note

In cantilever beam no need to calculate support reaction.

Shearforce calculation

Shearforce at B = $+W_1 = 1.5$ kN.

Shearforce at C = $1.5 + 2 = 3.5$ kN.

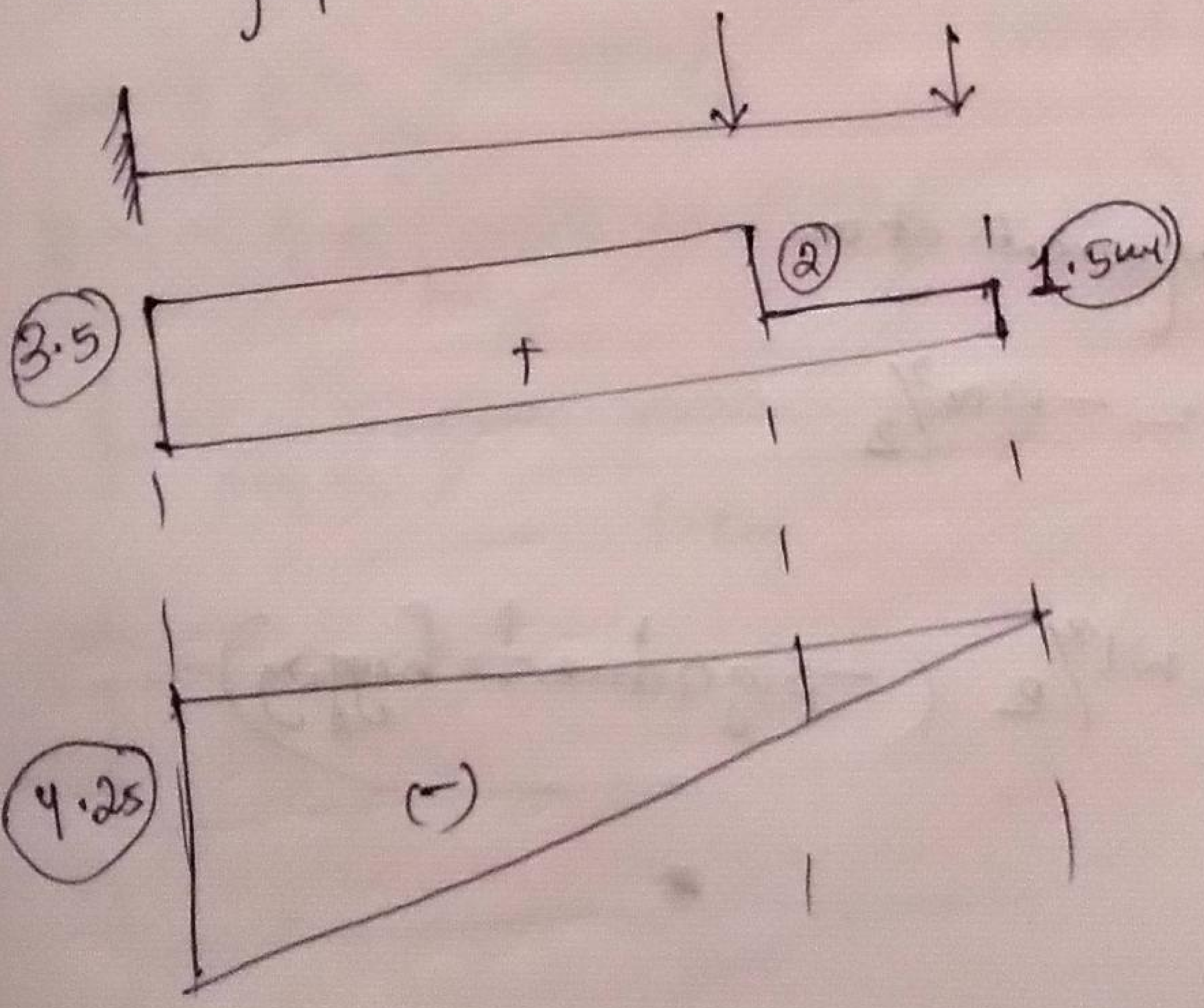
Shearforce at A = 3.5 kN.

Bending moment calculation

Bending moment at B = 0.

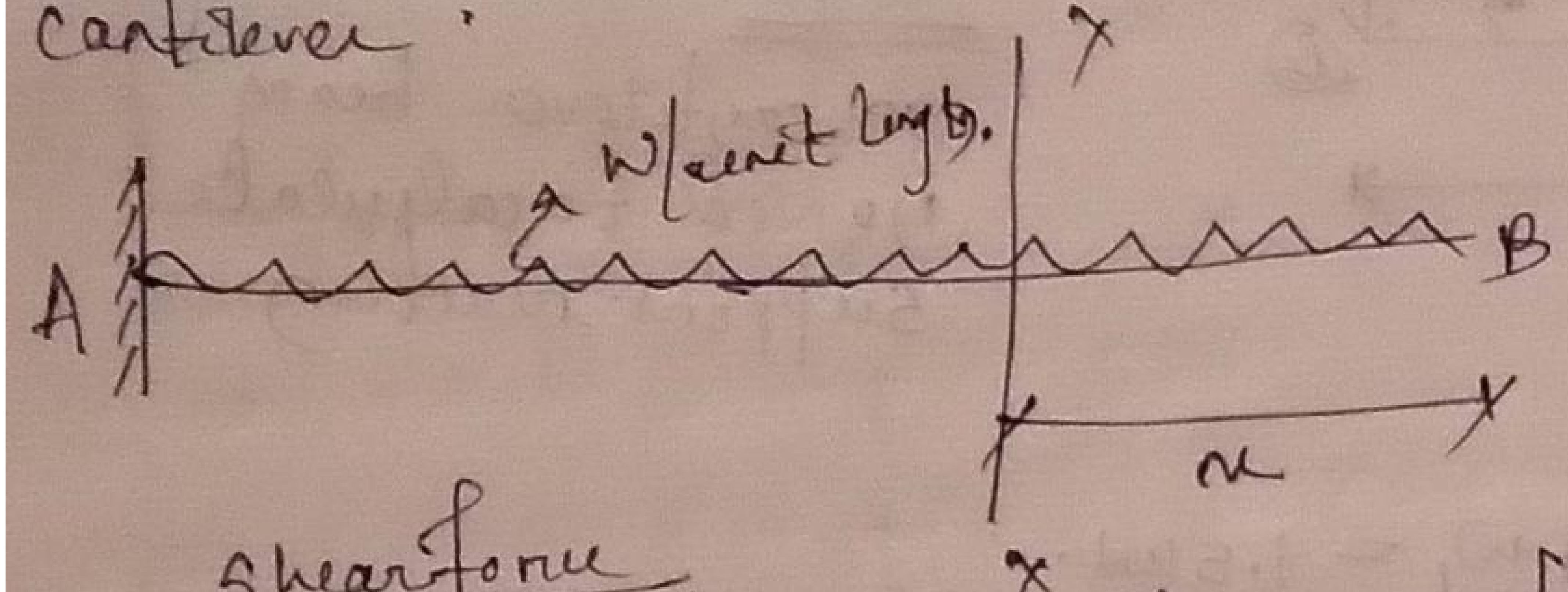
Bending moment at C = $-1.5 \times 0.5 = -0.75$ kNm

Bending moment at A = $(-1.5 \times 1.5) + (2 \times 1) = -4.25$ kNm.



* Cantilever with a udl :-

Consider a cantilever AB of length 'L' and carrying a udl of w/unit length over the entire length of the cantilever.



Shear force

We know that shear force at any section X at a distance x m from B.

S.f at $x = Wx$ (right downwards. +ve)

S.f at $A = 0$ (as $x = 0$) and increases to straight line WL at A.

at $x = L$

F_x or S.F at $x = W \times L$.

Bending moment

We know that Bending moment at x

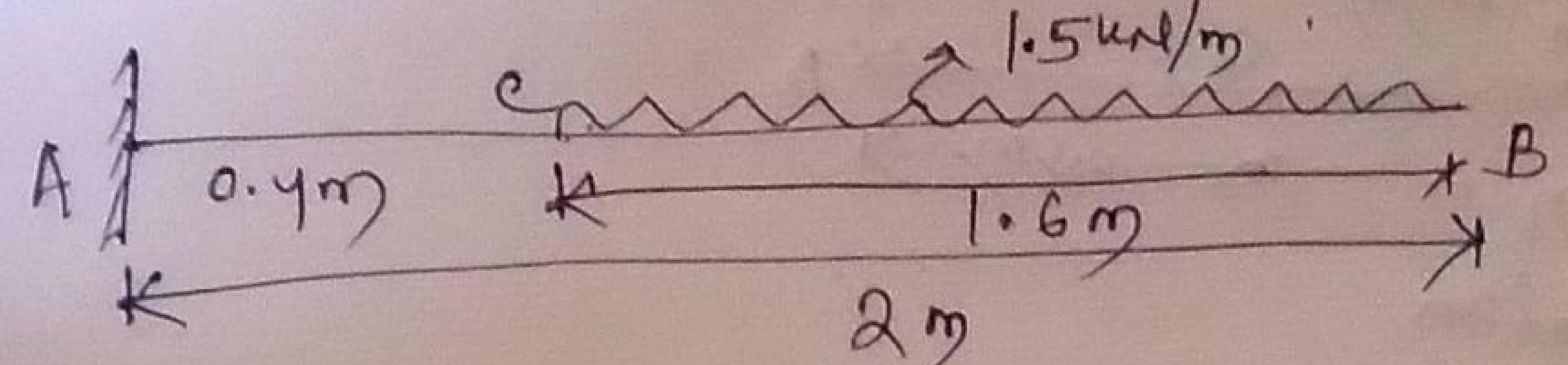
$M_x = -W \times x \times x/2 = -Wx^2/2$

When $x = L$

$\therefore M = -W \times L \times L/2 = WL^2/2$ (- sign due to hogging)

$\boxed{B.M_A = -WL^2/2}$.

Q. A cantilever beam AB, 2m long carries a udl of 1.5 kN/m over a length of 1.6 m from the free end. draw S.F and B.M.



Solution given data

Span(L) = 2m

udl = 1.5 kN/m

Shear force calculation :-

Shear force at B = 0

Shear force at C = $1.5 \times 1.6 = 2.4$ kN.

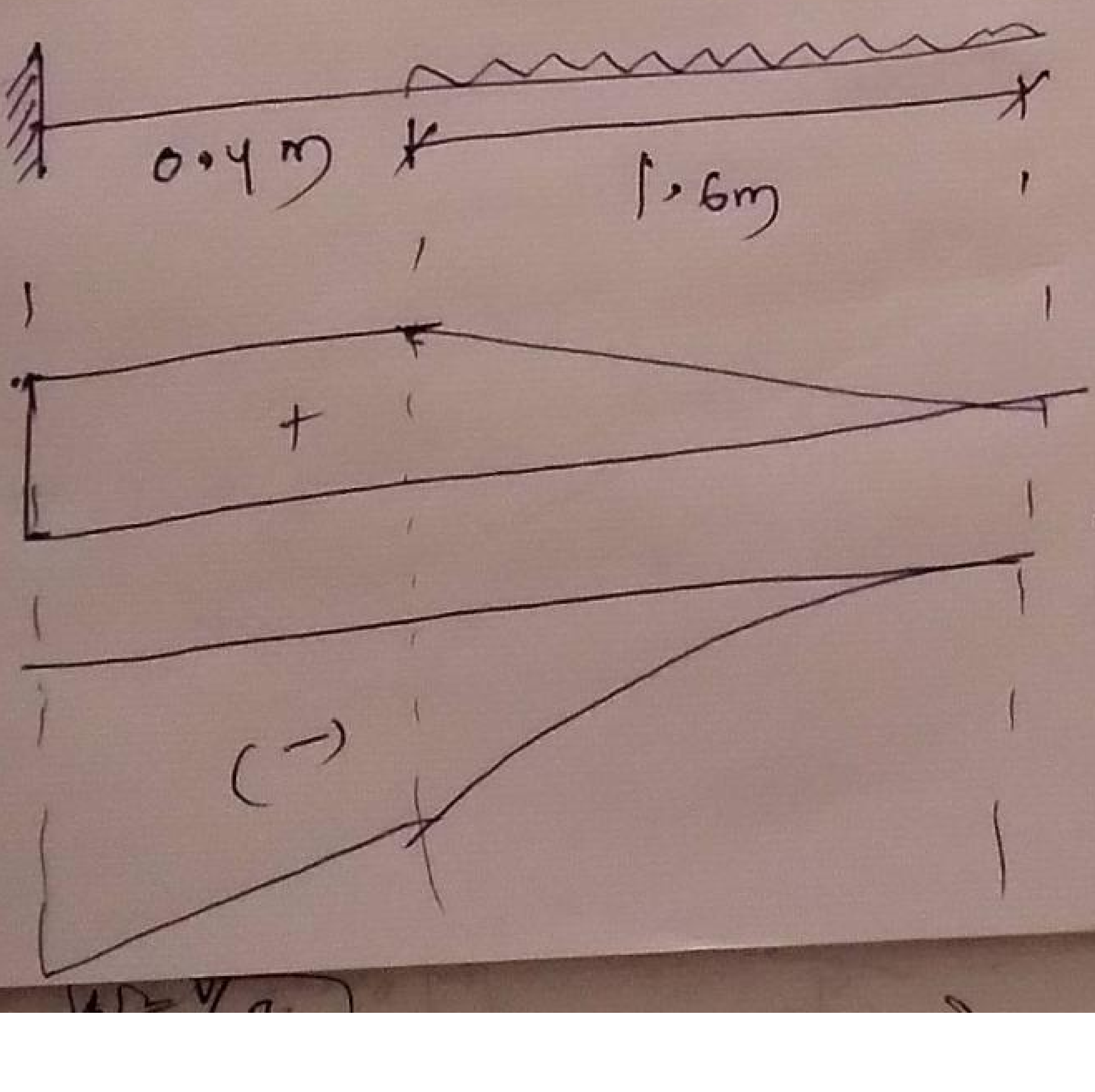
Shear force at A = 2.4 kN.

Bending moment calculation

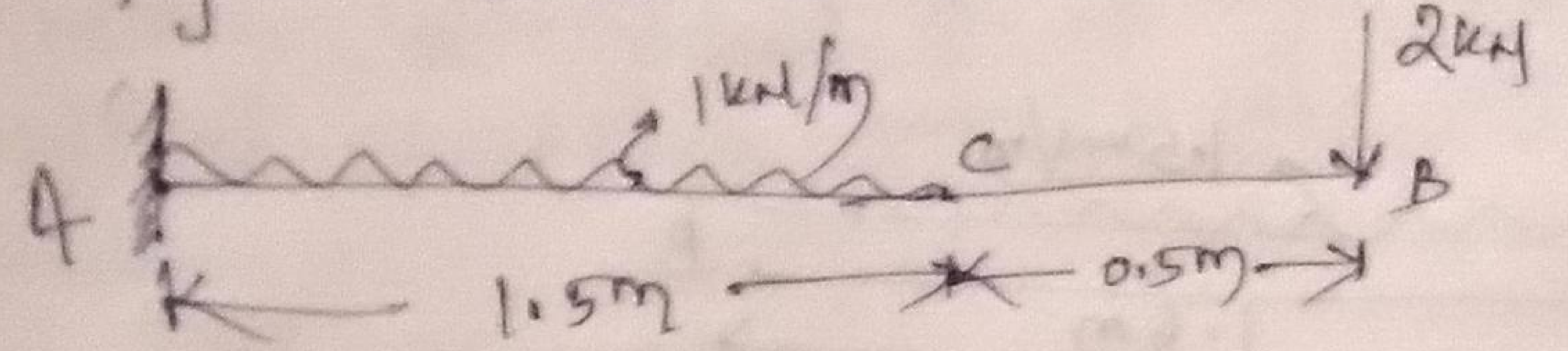
B.M at B = 0.

B.M at C = $w \times a \times a/2 = -1.5 \times 1.6 \times 1.6/2 = -1.92$.

B.M at A = $\left(-1.5 \times (1.6) \times \frac{1.6}{2} + 0.4\right) = 2.88$ kNm.



Q. A cantilever beam of 1.5m span is loaded as shown in figure. Calculate shear force and Bending moment.



Shear force calculation

Shear force at B = +W = 2kN.

Shear force at C = 2kN [(as no load between B and C)].

Shear force at A = $2 + (1\times1) = 3$ kN.

Bending moment calculation

B.M at B = 0.

B.M at C = $-2\times0.5 = -1$ kN/m.

B.M at A = $[(-2\times1.5) + (1\times1\times1/2)]$

$= -3.5$ kN/m.

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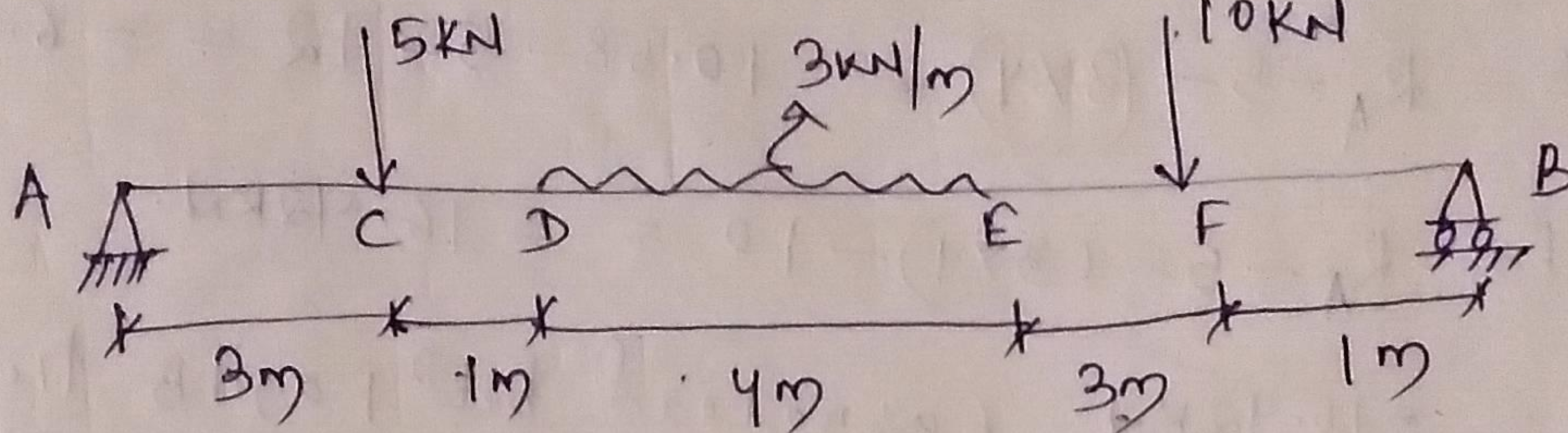
Q



Solution

Step-1:- Calculation of support Reaction

$\Sigma M_A = 0$

$\Rightarrow R_B \times 12 - 10 \times 11 - 3 \times 4 \times (4/2 + 4) - 5 \times 3 = 0$

$\Rightarrow R_B \times 12 - 110 - 72 - 15 = 0$

$\Rightarrow R_B \times 12 = 197$

$\Rightarrow \boxed{R_B = 16.42 \text{ KN}}$

upward force = downward force

$\Rightarrow R_A + R_B = 5 + (3\times4) + 10$

$\Rightarrow R_A + R_B = 27$

$\Rightarrow R_A + 16.42 = 27$

$\Rightarrow \boxed{R_A = 10.58\,KN}$

Step-2 : Calculation of shear force

Shear force at $A = +R_A = +10.58$ KN

S.F at $C = R_A - 5 = 10.58 - 5 = 5.58$ KN.

S.F at D = as no load betⁿ C and D S.F value will remain same as C = 5.58 KN.

S.F at $E = R_A - 5 - (3\times4) = 10.58 - 5 - 12 = -6.42$ KN.

S.F at $F = R_A - 5 - (3\times4) - 10 = -16.42$ KN.

S.F at B = -16.42 KN (as no load betⁿ F and B the S.F value will continue).

Step-3 : Bending Moment calculation

B.M at A = 0

B.M at $C = R_A \times 3 = 10.58 \times 3 = 31.74$ KNm.

B.M at $D = R_A \times 4 - 5\times1 = 10.58\times4 - 5 = 37.32$ KNm.

B.M at $E = R_A \times 8 - 5\times5 - 3\times4\times4/2 = 35.64$ KNm.

B.M at $F = R_A \times 11 - 5\times8 - 3\times4\times(4/2 + 3)$

$= 10.58\times11 - 40 - 75 = 16.38$ KNm.

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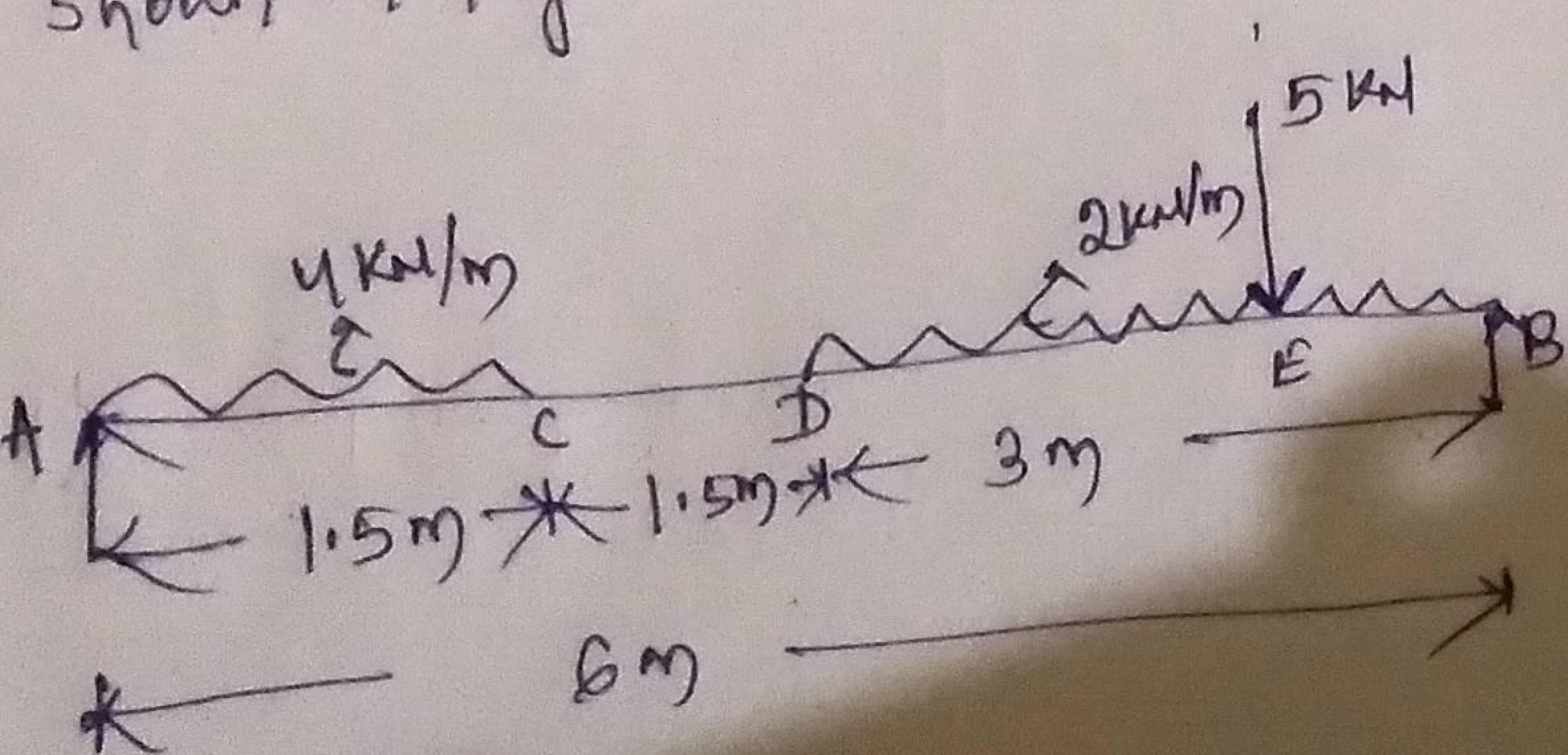
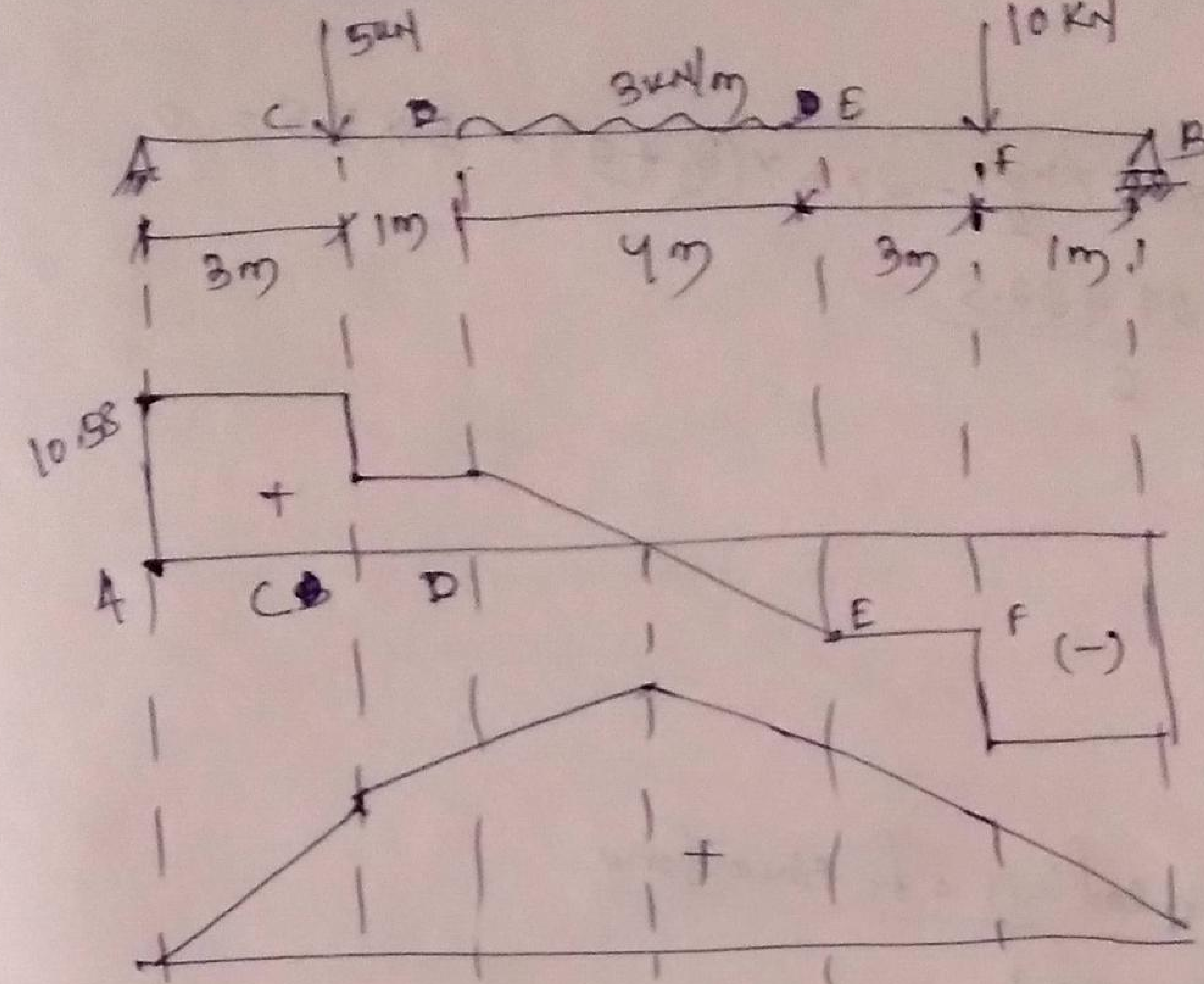
A simply supported beam AB, 6m Long is loaded as shown on figure. construct S.F & B.M and find value of Max B.M.



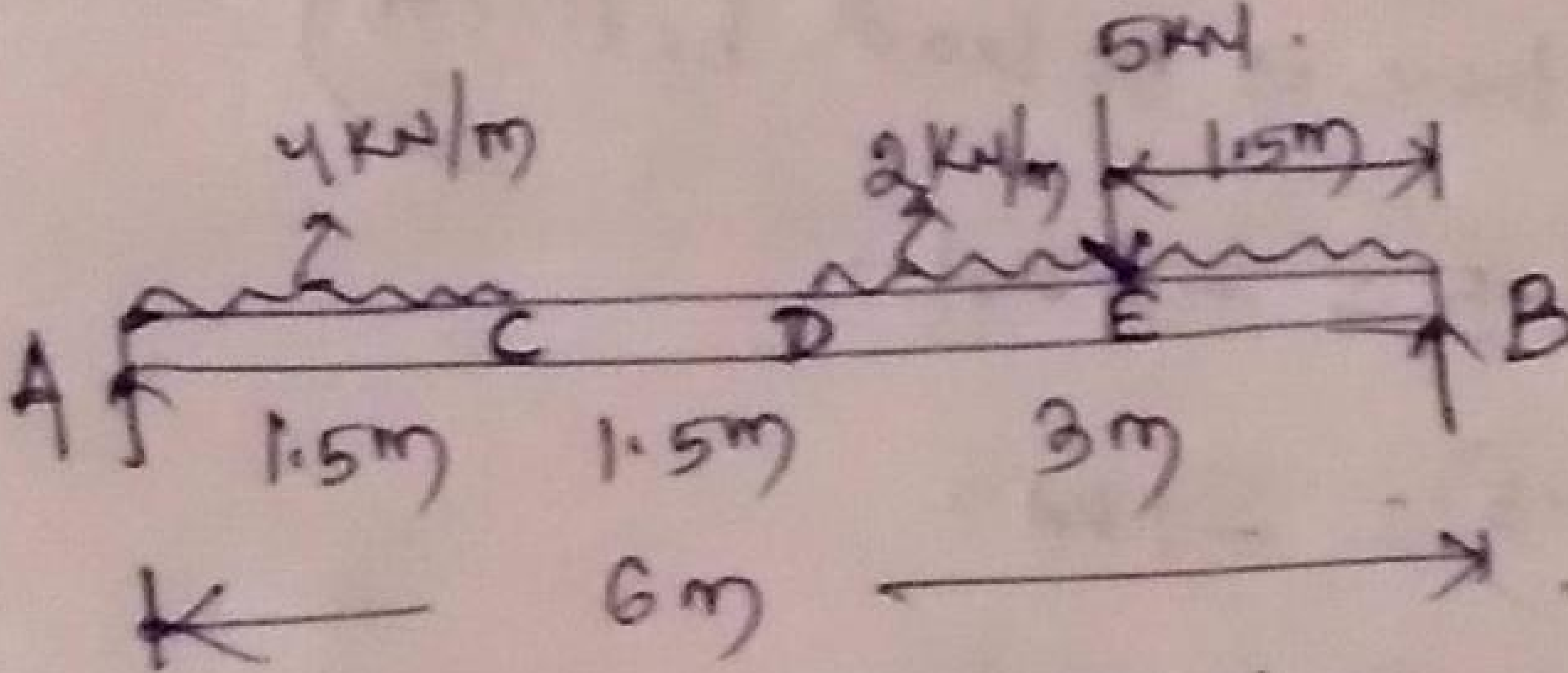
B.M at F $= R_A \times 12 - 5 \times 9 - 3 \times 4 \times \left(\frac{4}{2} + 4\right) - 10 \times 1$

$= 10.58 \times 12 - 45 - 12 \times 6 - 10 = 0$

Step-4:- SFD and BMD



Q. A simply supported beam AB, 6m long is loaded as shown in figure.



Construct the shear force and Bending Moment for beam & find the position and value of Max^m Bending Moment.

Ans:- given span (L) = 6m.

Step 1: Support Reaction

$R_A + R_B = 4 \times 1.5 + 2 \times 3 + 5$ (as we know upward force = downward force)

$\boxed{R_A + R_B = 17}$

$\Sigma M_A = 0$

$\Rightarrow R_B \times 6 - 4 \times 1.5 \times \frac{1.5}{2} - 2 \times 3 \times (3/2 + 3) - 5 \times 4.5 = 0$

$\Rightarrow R_B = \frac{4.5 + 27 + 22.5}{6}$

$\Rightarrow \boxed{R_B = 9}$ kN

$R_A + R_B = 17$

$\Rightarrow R_A + 9 = 17$

$\Rightarrow \boxed{R_A = 8}$ kN.

Step-2: Calculation of shear force

S.F at A = R_A = 8 kN

S.F at C = $R_A - (4 \times 1.5) = 2$ kN.

S.F at D = 2 kN (as there is no load bet.^{n} CD)

S.F at E = $2 - (2 \times 1.5) - 5 = -6$ kN.

S.F before E = $2 - (2 \times 1.5) = -1$ kN.

S.F at B = $-6 (-2 \times 1.5) = -9$ kN.

Step 3: Bending Moment calculation

B.M at A = 0

B.M @ C = $R_A \times 1.5 - 4 \times 1.5 \times 1.5/2 = 7.5$ kNm.

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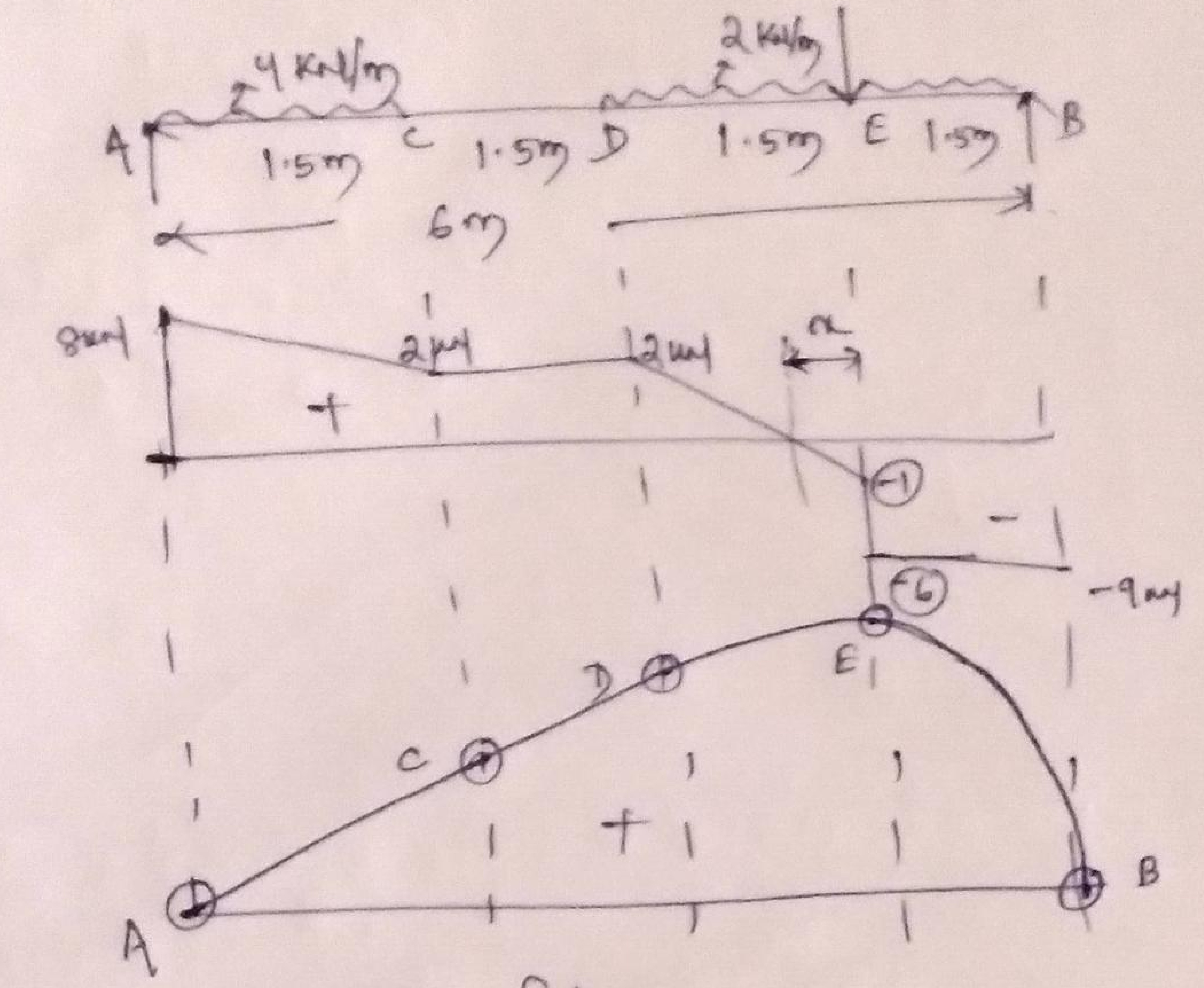
B.M at D $= R_A \times 3 - 4\times1.5\times\left(\frac{1.5}{2}+1.5\right)$

$= 10.5$ kN m.

B.M at E $= R_B \times 1.5 - 2\times1.5\times\frac{1.5}{2} = 11.25$ kN m.

B.M at B $= 0$.

We know Maximum B.M will occur where S.F changes sign.



from similar law of triangle we know that

$\frac{x}{1} = \frac{(1.5-x)}{2}$

$\Rightarrow 2x = 1.5 - x$

$\Rightarrow 3x = 1.5$

$\Rightarrow \boxed{x = 0.5\text{ m}}$

Maximum Bending moment

$M = 9\times(1.5+0.5) - (2\times2\times 2/2) - 5\times0.5$

$= \boxed{11.5\text{ kN m}}$